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CONTENTS

April, 1957

Papers

Journal of the
HYDRAULICS DIVISION
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MECHANICS OF SEDIMENT-RIPPLE FORMATION*

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(Proc. Paper 1197)

SYNOPSIS

Literature on the mechanics of sediment ripple formation is reviewed. It is shown in this paper that ripples are caused primarily by the instability of the zone of high velocity gradient at the surface of the sediment-laden bottom. For practical application an experimental criterion is given to predict the formation of sediment ripples and dunes.

INTRODUCTION

In an alluvial stream any appreciable amount of contact-load or bed-load movement is usually associated with sand ripples, sand bars, or sand dunes. Hydraulic engineers are well aware of the problem of the effect of such sand wave motion on sediment transport due to the uncertainty of its roughness. The reason sand ripples, bars, and dunes are associated with bed-load movement has not been explained satisfactorily. This problem is complicated because different sets of forces, such as viscous force and gravitational force, come into play at different stages of movement. This paper is not intended to furnish a complete answer to the problem because of the present limited knowledge of application of fluid mechanics to sediment transport. It is intended to discuss the fundamental mechanics of sand-wave formation and to give a possible physical explanation of this phenomenon. The proposed hypothesis is verified by laboratory experiments with the aid of dimensional analysis.

Like the problem of bed-load movement, bed configuration is also affected by different sets of forces at different stages of movement, and by the

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presence of external influences, such as rigid side boundaries and entrance conditions. However, since the undulation of a movable bed (in the form of either ripples, bars or dunes) is bound to occur regardless of the effects due to different kinds of forces at different stages of movement, it is obviously caused by some kind of motion inherent in the flow system. The result of such a motion is called sediment ripples in this paper. (The term "incipient dunes" has been suggested to the author by Professor E. W. Lane to denote this kind of sediment ripple, which eventually develops into dunes.)

Previous Investigations on Sand-Wave Formation

In 1883 G. H. Darwin(1) published in England his results of experiments on sand-ripple formation due to oscillatory motion. He stated that:

> "The ripple marks on a sand bed caused between two currents of fluid is dynamically unstable, but if a series of vortices be interpolated so as to form a function roller, as it were, it would probably become stable."

In the language of hydrodynamics, this means that there exists a vortex sheet between two parallel currents owing to a high velocity gradient. This vortex sheet is unstable in nature and eventually will break up into a series of vortices. Darwin did not explain the significance of such a vortex sheet, nor did he investigate the velocity gradient causing ripples.

Also in England, Bagnold(2) conducted extensive research on the formation of sand dunes caused by wind. He explained that desert dunes are due to the instability of the bed caused by the bombardment of sand grains.

Exner(3) established a differential erosion equation for two-dimensional flow:

$$\frac{\partial \eta}{\partial t} + K\frac{\partial v}{\partial x} = 0 \tag{1}$$

where η is the elevation of the bed relative to time,
- t is time,
- K is a factor relating sediment discharge to flow velocity,
- v is the flow velocity near the bed and is a function of distance along the bed, and
- x is the distance in the downstream direction.

This equation indicates that the change in bed elevation is due to longitudinal variation of bottom velocity. Using this equation, Exner showed that if a symmetrical sand wave is taken as an initial bottom surface it will be transformed in course of time into an asymmetrical wave with a gentle upper slope and a steep lower slope, which corresponds to experimental data. However, he did not give the reason why there can be such a longitudinal variation of velocity if the bottom is originally plane.

Anderson(4) reasoned that in case of shallow flow, surface waves may affect the alluvial bottom and cause sand waves. Considering the interaction between surface waves and sand waves, and using Exner's equation of erosion, he obtained a relationship between the Froude number and the depth-wave length ratio. His explanation of initial ripple occurrence is as follows:

> "The periodic nature of waves that form on the bed suggests that the variation in the velocity that creates the waves is also periodic, but to

have a wave originate on an initially smooth bed requires that the velocity variations be initially due to a source apart from the forced wave shape—we may consider then a flow pattern in which a hypothetical surface wave exists on the flow over a smooth bed."

According to Anderson, the periodic variation of velocity with respect to distance is usually reflected in the surface waves. Such a variation of velocity distribution can be represented approximately by the potential function of surface waves. When there is no free surface, the variation of velocity distribution still exists, but it cannot be expressed by a potential function of surface waves. However, Anderson did not explain the cause of such periodic variation of velocity along the bed, which transforms a smooth bed into a rippled one. He did not explain the case of deep flow having sand ripples along the bed or desert dune formation without a free surface.

The premise that ripple formation is due to turbulence is favored by some hydraulic engineers, such as Velikanove(5) and Tison.(6) Velikanove was able to prove mathematically that turbulence could cause erosion and deposition along the bottom. Tison found that ripples were present in turbulent flow only. However, this theory leaves the following questions unexplained:

a) Since turbulence is non-periodic, how can it produce regular sand wave patterns?

b) What is the relation between the scale of turbulence and sand wave form, especially for the case of ripples formed on the upstream faces of large sand waves?

c) What is the effect of turbulence on sand ripples in case the laminar sublayer exists along the bottom?

d) Since sediment ripples and waves can create turbulence, how can it be proved that turbulence causes ripples and not vice versa?

Von Kármán(7) gave two methods of approach to express the sand wave length by assuming that:

a) it is related to the velocity of sand particles passing over an existing ridge, and

b) the flow velocity along the wavy surface is constant.

It seems that both these approaches are not entirely sound because:

a) experiments indicate that a small ridge on an alluvial bottom may be smoothed out by the flow, while a large ridge should be considered as an artificial means to create sand waves, and

b) the flow velocity along the wavy surface does not seem to be constant, at least in the case of water flowing over a wavy bed.

C. C. Inglis(8) explained ripple formation in the following way:

"As the sand grains in a given mixture vary in size and shape; and in turbulent flow, any random collection of particles will occur on the bed—this will cause a small eddy to form downstream; and, due to the formation of this eddy, material will be scoured from the trough and the scoured material will deposit at a point further downstream to form another ripple crest which, in turn, will lead to the formation of another

eddy; and the process thus becomes continuous downstream until a series of ripple crests and troughs is formed."

This explanation is also doubtful because such random collection of particles generally changes the bed elevation by the magnitude of the particle size, and causes small eddies of the order of the particle size. This is too small compared with the ripple size found in nature. Furthermore, it was found through experiments that small irregularities of bed sometimes can be damped out by the flow.

Previous Investigations on Interface Instability

If it can be assumed that the sediment-ripple formation may be related to a problem of instability of:

a) generating waves on a surface of discontinuity, and

b) laminar layer due to viscosity effects near the movable boundary.

Instability of Surface of Discontinuity

In the case of two superposed fluids of densities ρ_1 and ρ_2 moving at velocities U_1 and U_2, conditions of stability are well known[9] (Fig. 1). The interface between the two fluids is a surface of discontinuity with respect to both the velocity and the density. The interface will be unstable if

$$(U_1-U_2)^2 > \frac{g\lambda}{2\pi} \frac{\rho_2^2-\rho_1^2}{\rho_1 \rho_2} \qquad (2)$$

where λ is wave-length. In Eq. 2 it is assumed that:

a) the flow is two-dimensional,

b) velocity of the flow is along one direction only,

c) the fluid is non-viscous,

d) no turbulence is present,

e) both fluids have infinite depth, and

f) gravitational force alone is considered.

In the case of $\rho_1 = \rho_2$, i.e., the surface of discontinuity is due to the velocity difference alone, such an interface is unstable for all wave lengths. The result is that the surface of discontinuity breaks up into a larger number of vortices. This instability was first stated by Helmholtz.[9] A physical explanation of the breaking up of the surface of discontinuity was given by Prandtl[10] as well as Rouse[11] (Fig. 2). Owing to fluctuations in the flow, the surface of discontinuity may take on a slightly wavy form as shown in Fig. 2(b). According to the equation of continuity between two streamlines, the local velocity decreases as the neighboring streamlines diverge and increases as they converge. According to Bernoulli's theorem, as the velocity decreases, the pressure intensity must increase and vice versa. Consequently, there is an increase in pressure (indicated by + signs) where the streamlines are further apart and a decrease in pressure (indicated by - signs) at the troughs where the streamlines are closer together. This

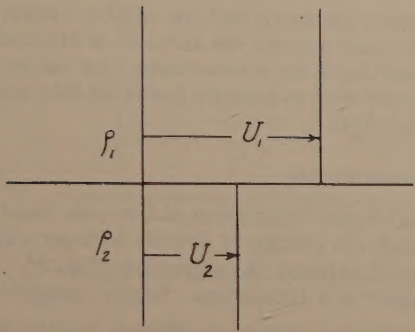

Fig. 1 Definition Sketch for Two Superposed Fluids.

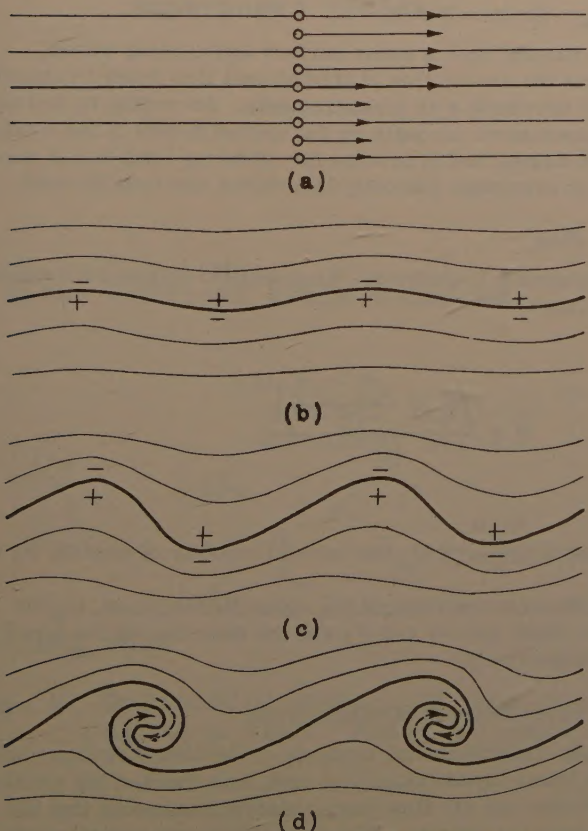

Fig. 2 Instability of Flow at a Surface of Discontinuity.

distribution of pressure clearly indicates that the motion cannot possibly be steady, as the pressure difference across the surface of discontinuity tends to further increase the distortion of the streamlines. As the process continues, the zone of discontinuity will eventually break up into separate eddies of finite size as shown in Fig. 2(d).

Instability of Three Superposed Fluids

Taylor(12) and Goldstein(13) gave a solution of the case involving three layers of superposed streams; the change of density between each layer is discontinuous but the change of velocity is continuous (Fig. 3). The velocity distribution in the middle layer is a transition. Taylor remarked that:

a) a velocity transition when the density is uniform causes long waves to be unstable, and

b) a discontinuity in density when there is no velocity transition makes the long waves stable while the short waves unstable.

He presented an equation to determine stable wave lengths for the case when both velocity transition and density discontinuity are present.

Wind-Generated Water-Surface Waves

In the case of air flowing over a water surface and causing waves, Jeffreys(14) found that the assumption of irrotational flow leads to unsound conclusions which do not check with measurements. According to Jeffreys' theory, the wind presses more strongly on the upwind slopes of the waves than on the sheltered slopes, and it is when the resulting tendency of waves to grow is just able to overcome viscosity that waves are first formed.

Mixing of Stratified Flow

Using a similar approach to Jeffreys', Keulegan(15) obtained a criterion of mixing of stratified flows:

$$\theta = \frac{\left(\nu_2\, g\, \frac{\rho_2 - \rho_1}{\rho_1}\right)^{1/3}}{U_1} \tag{3}$$

where θ is a function of $\left(\frac{U_c R_1}{\nu_1}\right)$, U_c the critical velocity of mixing, R_1 the hydraulic radius of the cross-section of the upper flowing fluid, U_1 the velocity of the upper fluid, and ρ_1 and ρ_2 are the densities of the upper and the lower liquids, respectively.

Laminar Boundary Layer Between Stratified Flow

Keulegan(16) made a mathematical analysis of the boundary layer between two liquids with the lower liquid initially at rest, and checked his results with those obtained by Dryden for air flow over a plate by assuming that the viscosity of the lower liquid is infinite. The comparison was satisfactory.

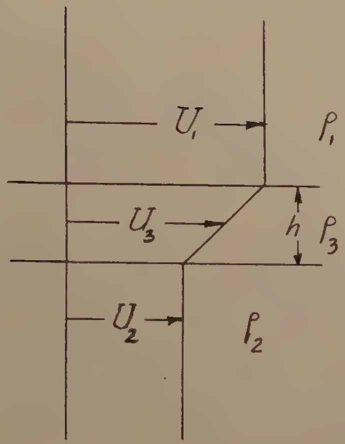

Fig. 3 Definition Sketch for Three Superposed Fluids.

Note: Vertical scale enlarged

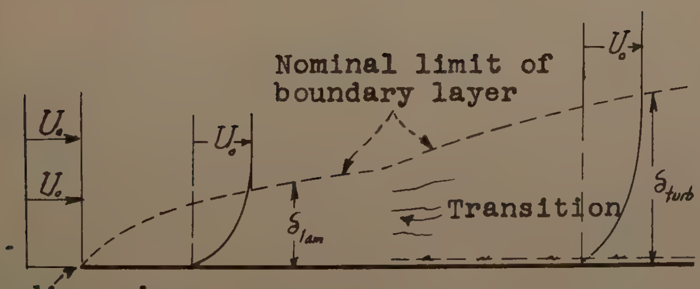

Fig. 4 Definition sketch of the Boundary Layer along a Flat Plate at Zero Incidence.

Instability of Laminar Boundary Layer

Since Prandtl introduced the boundary layer concept into the science of fluid mechanics in 1904, considerable research work has been done on this subject. Most of the work has been on flow over a rigid boundary.

One of the important problems concerning the boundary layer is its transition from laminar to turbulent flow. Before the mechanics of transition is discussed, it may be appropriate to review briefly the boundary layer theory.

When any fluid flows past solid boundaries, the fluid near the boundary is retarded by friction. This retarded layer is known as the boundary layer. Figure 4 shows parallel flow past a flat plate. At any arbitrary section along the plate, the velocity varies nearly parabolically, having a magnitude of zero at the plate and approaching U_0 with normal distance δ. Usually δ is called the boundary layer thickness, and is arbitrarily defined as that distance from the boundary where the velocity differs by 1 per cent from U_0. It decreases with increasing velocity, and increases with increasing kinematic viscosity. For a given fluid, a given plate, and a given velocity of flow, it increases with increasing distance from the leading edge owing to continual action of the frictional forces. As the layer grows in thickness, it becomes unstable and eventually changes from laminar to turbulent. Then it is called turbulent boundary layer. Within the turbulent boundary layer, there is a very thin layer next to the boundary that still has laminar motion. This is called the laminar sub-layer. The natures of these three boundary layers are entirely different.

The mechanics of transition from laminar to turbulent boundary layer had not been known until the stability theory by Tolimien and Schlichting was proved experimentally by Schubauer and Skramstad.(17) According to this theory, small disturbances within a certain range of frequency and wave length are amplified, whereas disturbances outside this range are damped, provided that the Reynolds number exceeds a certain limiting value. It is further assumed that the amplification of disturbances eventually affects the transition from laminar to turbulent flow. The following is quoted from a statement by Schubauer and Skramstad.

> "It is not difficult to imagine a process here like that often assumed for the formation of eddies from a free vortex sheet. The sheet is imagined to take first a wave-like character, then as the wave grows, to curl up into discrete eddies. The disturbed laminar boundary layer may be regarded as a wavy vortex layer with the wave progressively increasing in amplitude and distorting until discrete eddies are formed. The eddies themselves are unstable and soon break up into a diffusive type of motion which characterizes turbulent flow.
>
> "The boundary layer oscillations may arise from internal disturbances as well as from external disturbances—that is, from surface irregularities and vibration of the surface. A randomly distributed small roughness may produce effects similar to small amounts of turbulence in the air stream. Small ridges of waves in the surface may start oscillations when the spacing is near some amplified wave length. Vibration of the surface, like sound, may produce oscillations, especially when the frequency is near some amplified oscillation frequency."

Effect of Roughness on Instability of Laminar Boundary Layer

There are in general two schools of thought in regard to the effect of roughness on transition from laminar to turbulent flow. An early physical concept is that a roughness element induces transition when the Reynolds number of the element reaches a definite critical value at which vortices appear. This was favored by Schiller. A second concept according to Dryden(18) is

> " . . . . arises from discussions in the literature of roughness effects in pipes, in which it is frequently stated that the effect is dependent on the height of the roughness element in relation to the boundary layer thickness. In addition to the intuitive appeal of this concept, some support is provided by the Tollmien—Schlicting theory of boundary layer instability. The roughness element imposes a disturbance that is added to those already present from the initial turbulence of the stream. Its contribution to the total disturbance within the critical wave-length range for which amplification occurs, depends on its shape and its height relative to the boundary layer thickness. Its presence produces earlier transition, since with a large disturbance less amplification is required. If all disturbances are small, we might expect the roughness effect to be independent of other disturbances present regardless of their source."

Despite the lack of definite conclusions as to the effect of roughness on transition from laminar to turbulent flow, for practical purposes, Schlicting(19) uses Schiller's concept and chooses for critical Reynolds number

$$\frac{U_* d}{\nu} = \text{Constant}$$

where U_* is the shear velocity and d is roughness, to predict the transition from laminar to turbulent flow.

Application of Instability Theory to the Problem of Sediment-Ripple Formation

The assumption that a sediment-laden bed can be considered as a fluid is justified because

a) from Keulegan's analysis, a rigid boundary can be treated as a fluid having infinite viscosity, and

b) sediment density current is usually accepted as being a fluid, and sediment-laden bed is an extreme case of a density current.

If Keulegan's experiments on stratified flow are compared with laboratory experiments on flow over an alluvial bed, they are very much similar to each other. For example, in an experiment by Keulegan, the flow took place in a closed rectangular channel with a lower pool of liquid at rest, the upper liquid was fresh water, the lower liquid salt or sugar solution. According to Keulegan, at low velocities a smooth interface was discernible even when the lower liquid was not colored. At this initial stage, observations showed that

at the interface, and in a finite band on the two sides of the interface, the flow was laminar even when the central regions of the upper current were eddying or turbulent. In the case of flow over an alluvial bed, this means there may exist a laminar boundary layer or laminar sub-layer close to the boundary, even if the main flow is turbulent. Keulegan also found that when the velocity of the current was increased to some definite value, depending on the densities and the viscosities of the two liquids, the interface appeared to be covered with ridges extending from one glass wall to the other. These ridges moved progressively downstream, and their crest heights were greatest at the center. This is also applicable qualitatively to the generation of sediment ripples. With further increase of the velocity of the current, the waves became sharp crested and mixing commenced. This action is very similar to the phenomenon of sand dune movement and the early stage of saltation movement.

Keulegan noticed that increased velocities caused the height of the waves to increase but did not substantially affect the length of the interfacial waves. Straub[20] found that the sediment wave length was also not affected substantially by velocity increases.

It is important to note that Keulegan found a laminar boundary layer between the two stratified flows, even though the central portion of the flow was turbulent. In the case of flow over an alluvial bed, it means there is usually a laminar zone next to the bed even though the main flow is turbulent. At the entrance, such a laminar zone is the laminar boundary layer and it develops into a turbulent boundary layer having a laminar sub-layer as the flow proceeds downstream.

The viscosity effect is dominant within the laminar zone. The flow outside laminar zone can be considered as irrotational flow. If the laminar zone, which is usually very thin for water or air, is considered as an interface between the superposed fluid and the alluvial bed, the problem of sediment-ripple formation will become a problem of interface instability. The presence of a thin laminar zone between the flow and the alluvial bed will change considerably the classical approach of Helmholtz and Jeffreys.

Although it is reasonable to assume that the phenomenon of sediment-ripple formation can be explained by the theory of instability, it is difficult to find a direct solution to the problems of sediment-ripple formation because present knowledge of the instability theory is still very limited and is based upon special boundary conditions such as two-dimensional flow near a rigid boundary.

Based upon the present knowledge of instability theory, the phenomenon of sediment-ripple formation can be pictured as follows: if the alluvial bed is considered to be a fluid of unknown depth, having unknown density and viscosity, flow over such a bed can be treated theoretically as a problem of stratified flow having an interface, within which the viscosity effect causes the velocity to vary from zero at the bed to that of the stream flow.

In case the Reynolds number of the flow is very high, the viscosity effect is very small and the interface approaches a vortex sheet (or vortex layer). The generation of sediment ripples can then be comprehended through the approach of the Helmholtz-Jeffreys' instability criterion.

In case the main flow is turbulent and the interface has a finite thickness in the form of a laminar sub-layer, the velocity fluctuation in the main flow cannot effectively penetrate into the laminar sub-layer zone if $\frac{U_* d}{\nu} \leq 4$.

Sediment ripples found under this condition cannot be caused by turbulence but can be attributed to instability of the laminar sublayer.

In case the interface is in the form of laminar boundary layer, such as is found at an inlet transition, sediment ripples are caused by the instability of laminar boundary layer. The fact that turbulence follows the breaking down of laminar boundary layer just like sediment ripples do may give rise to the erroneous belief that turbulence is the cause of sediment ripples.

The problem of instability of a laminar flow due to the presence of sediment particles is very complex. It may be possible, however, from a dimensional analysis point of view, that a dimensionless parameter such as $\frac{U_* d}{\nu}$ can be used to indicate the instability of the laminar boundary layer, and that the laminar sub-layer can be defined as an interface in this method of analysis.

Criterion of Sediment-Ripple Formation

Use of the instability theory to explain the phenomenon of sediment-ripple formation has been discussed above. The amplified oscillatory motion at the interface can cause periodic variations of velocity along the bottom because, according to the principle of continuity between two streamlines, the local velocity will decrease as the neighboring streamlines diverge, and increase as they converge. If the local velocity of the flow is increased sufficiently to transport the sediment, then according to Exner's equation

$$\frac{\partial \eta}{\partial t} + K\frac{\partial v}{\partial x} = 0 \tag{1}$$

the periodic changes in velocity along the bed will result in a change of bed configuration. The conditions at the moment sediment ripples appear on the bed must be:

a) the flow is able to transport the sediment, and

b) the interface between the flow and the movable bed has become unstable.

Two dimensionless parameters for the above two conditions are obtained as discussed below.

Scouring Force of the Flow and Resistance of Sediment Particles

The scouring force acting on a submerged particle can be expressed as

$$F_1 = C_D \frac{\pi}{4} d^2 \frac{1}{2} \rho v^2 \tag{4}$$

where C_D is a drag coefficient, a function of $\frac{U_* d}{\nu}$ and shape factor of the particle; d the size of the particle; v the velocity flow at the particle level; and ρ the density of the fluid.

The flow velocity at the particle level can be written in terms of shear velocity according to the Kármán-Prandtl velocity equation

$$\frac{v}{U_*} = \phi\left(\frac{U_* d}{\nu}\right) \tag{5}$$

Equation 5 is a functional form for flow near a smooth boundary. Substituting Eq. 5 into Eq. 4, one obtains

$$F_1 = C_1 \frac{1}{4}\pi d^2 \frac{1}{2}\rho U_*^2 \tag{6}$$

where C_1 is also a function of $\frac{U_* d}{\nu}$ and shape factor of the particle. Equation 6 indicates that the scouring force of the flow is a function of the shear velocity, projected area of the sediment particle, density of the fluid, Reynolds number and shape factor of the sediment particle.

The resistance of the sediment to the flow is usually considered to be a function of particle size, particle shape, and submerged weight of the particle. Experiments by Krumbein[21] indicate that sediment fall velocity is a very significant parameter in the flume behavior of a particle. Einstein[22] introduces fall velocity parameter into his bed-load function through the use of dimensional analysis. Kalinske[23] also indicated the possibility of fall velocity being an important parameter for bed-load movement. How the fall velocity affects the bed-load movement is not yet known. Some explanation was given by Quraishy[24] and Liu,[25] but further research is needed. So far, experiments indicate that fall velocity is an important parameter in the study of bed-load movement. The resistance of the particle can be expressed in terms of its fall velocity:

$$F_2 = C_2 \frac{1}{4}\pi d^2 \rho \frac{w^2}{2} \tag{7}$$

where w is the fall velocity of the sediment particle and C_2 a function of $\frac{\Delta v d}{\nu}$ in which Δv is the velocity of the particle relative to its surrounding fluid.

Since Δv can be intuitively assumed to be a function of local flow velocity, which is a function of U_*, C_2 may be a function of $\frac{U_* d}{\nu}$ and shape factor.

Laboratory experiments indicate that ripples occur shortly after the beginning of bed-load movement; therefore it can be assumed without serious error that at the moment ripples occur, the scouring force is equal to the critical scouring force, i.e.,

$$F_1 = F_2$$

or

$$C_2 \frac{1}{4}\pi d^2 \frac{1}{2}\rho w^2 = C_1 \frac{1}{4}\pi d^2 \frac{1}{2}\rho U_*^2 \tag{8}$$

Simplifying Eq. 8 yields

$$\frac{U_*}{w} = \phi_1\left(\frac{U_* d}{\nu},\ \text{particle shape factor}\right) \tag{9}$$

If the shape factor is of secondary importance and can be omitted, the resulting expression is

$$\frac{U_*}{w} = \phi_2\left(\frac{U_* d}{\nu}\right) \tag{10}$$

The term $\frac{U_*}{w}$ may be called movability number of the sediment particle.

Index of Interface Instability

Although it is difficult to obtain a mathematical solution to predict the interface instability as discussed previously, it is still possible to find a dimensionless parameter to indicate the interface instability by physical reasoning similar to the X-number adopted by Rouse.(11) Such an instability index is chosen as $\frac{U_* d}{\nu}$, which is a special form of Reynolds number. It also indicates the magnitude of the sediment particle relative to the thickness of the laminar boundary layer. On the one hand, the greater the shear velocity (which means the higher the velocity gradient at the interface), the more pronounced is the instability of the interface; and the larger the sediment, the larger the disturbances it causes, and therefore the less the amplification required for the interfacial waves to become unstable. On the other hand, the higher the viscosity, the more the internal shear tends to oppose further differences in velocity, and also to dissipate the energy of oscillation motion.

Physically, it is expected that the instability index $\frac{U_* d}{\nu}$ is not a constant but varies with the movability number of the sediment. Since both parameters $\frac{U_*}{w}$ and $\frac{U_* d}{\nu}$ are included in Eq. 10, it can be considered as the general equation of criterion for sediment-ripple formation. It should be noted that $\frac{U_* d}{\nu}$ has a twofold meaning:

a) It correlates the flow velocity at the grain level to the shear velocity in accordance with Kármán-Prandtl theory, thereby defining the scouring force; and

b) it indicates the instability of the interface.

Experiments on the Beginning of Sediment-Ripple Formation

It is difficult to define the beginning of sediment ripples experimentally. In order to determine accurately the conditions of ripple formation, the initial bed condition must be uniform and plane. Non-uniform distribution of the flow due to poor preparation of the bed will cause concentration of flow, which might result in local sediment movement and ripples. Marked bed changes will cause earlier formation of ripples. However, experiments have indicated that initial small ridges and undulations of the bed are un-

important to the formation of ripples. The velocity variations caused by small ridges soon vanish owing to the viscous effect.

In the experimentation, a recirculating, open-channel, laboratory flume 40 ft. long, 1 ft. wide, and 2 ft. deep was used. There were glass-walled sections on both sides of the flume to facilitate observation. Water was supplied through a centrifugal pump to a 4-in. feed pipe. Flow was measured through an orifice meter and returned to the supply sump over an adjustable tailgate at the downstream end of the flume. Two types of sand were used in the tests: Missouri River sand (d_{50} = 0.2 mm) and Ottawa 20 - 30 sand (d_{50} = 0.69 mm). The purpose of these tests was to verify a criterion established from available data, therefore only a few tests were needed. Results are tabulated in Table I.

During the experiments, special attention was given to determining whether sediment ripples are caused by surface waves. Two different kinds of experiments were made for this purpose. One test was made with the water surface covered by a board to eliminate the surface waves; severe sand waves were found in this case (Fig. 5). Another test was made with a flow having a low Froude number, $\frac{V}{\sqrt{gy}} = 0.22$. This flow was classified as a deep flow, because the depth (0.387 ft.) was greater than one-half the surface wave length (0.2 ft.). According to the principles of hydrodynamics, surface waves have no effect on the bottom for deep flow. Under this condition, sand waves were also found. In another test, which was not recorded, there was no visible surface wave; the flow was very calm and it was difficult to notice sand ridges along the bottom. It was found later, after the flume had been gradually drained, that there were small ridges or sand waves (Fig. 6). It is concluded therefore, that surface waves are not the primary cause of sediment ripples.

It was mentioned previously that it is difficult to make up a plane, uniform bed. This difficulty was overcome by the following technique: A test section was chosen about 8 ft. downstream from the entrance and 10 ft. above the tailgate. Transitions at both ends were made by lowering the sand bed (Fig. 7). The slope of the bed in the test section was obtained by placing excessive sand upstream and letting the flow wash the sand into a sloping bed. After the bed had been thus prepared, water was admitted from the downstream end until the entire test section was well submerged. Then the flow was introduced gradually into the flume by opening the upstream valve. The discharge was increased in very small increments. Data taken after the first appearance of sand waves were considered as pertaining to the beginning of sand waves. It was noticed during these tests that sediment ripples appeared suddenly on the bed at certain flow conditions (Fig. 8). Depth of flow was measured by a point-gage. In most cases, discharge was not measured because it was too small for the existing orifice meter.

Discussion of Experimental Results

The author's data, together with those obtained from other sources, are plotted in Fig. 9 in accordance with Eq. 10. The data from other sources are the average between smooth bed and first appearance of ripples because the large increments of discharge precluded determination of the exact conditions under which ripples first formed.

In general the data clearly define an experimental criterion of sediment-

TABLE I

Summary of Experimental Results at the Beginning of Ripples

Run No.	1-1	1-2	1-3	1-4**	1-5	1-6
Sediment	Mo.R.S.	Mo.R.S.	Mo.R.S.	Mo.R.S.	Ottawa Sand	
diameter mm	0.2	0.2	0.2	0.2	0.69	0.69
depth ft	0.0142	0.0117	0.0105	0.387	0.037	0.040
slope	0.0067	0.0070	0.008	0.000196	0.0050	0.00546
T lb/ft^2	0.0059	0.0051	0.0052	0.0047	0.0115	0.0131
U_* fps	0.055	0.051	0.052	0.049	0.077	0.082
t °F	60	69	69	61	69	65
x 10^5 ft^2/sec	1.21	1.066	1.066	1.094	1.066	1.125
w fps	0.074	0.082	0.082	0.074	0.361	0.361
$\frac{U_*}{w}$	0.74	0.62	0.635	0.73	0.21	0.23
$\frac{T}{(\ _s - \ )d}$	0.091	0.079	0.080	0.073	0.050	0.057
$\frac{U_* d}{\ }$	3.0	3.1	3.2	3.0	16.4	16.3

** Length of surface waves 0.2 ft to 0.18 ft.
V = 0.775 fps.

Fig. 5 Photograph of Bed Formation under Condition of Water Surface with a Board. Ottawa 20-30 sand, Q = 0.4 cfs, G = 38.3 lbs/hr.

Fig. 6 Photograph of Sand Wave very small Amplitude.

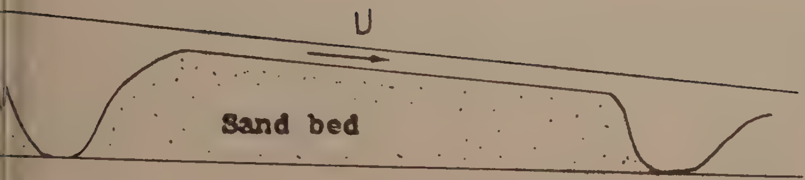

Fig. 7 Schematic Diagram of Flow over a Sand Bed.

Fig. 8 Photograph of First Appearance of Ripples, Run Nos. 1-3.

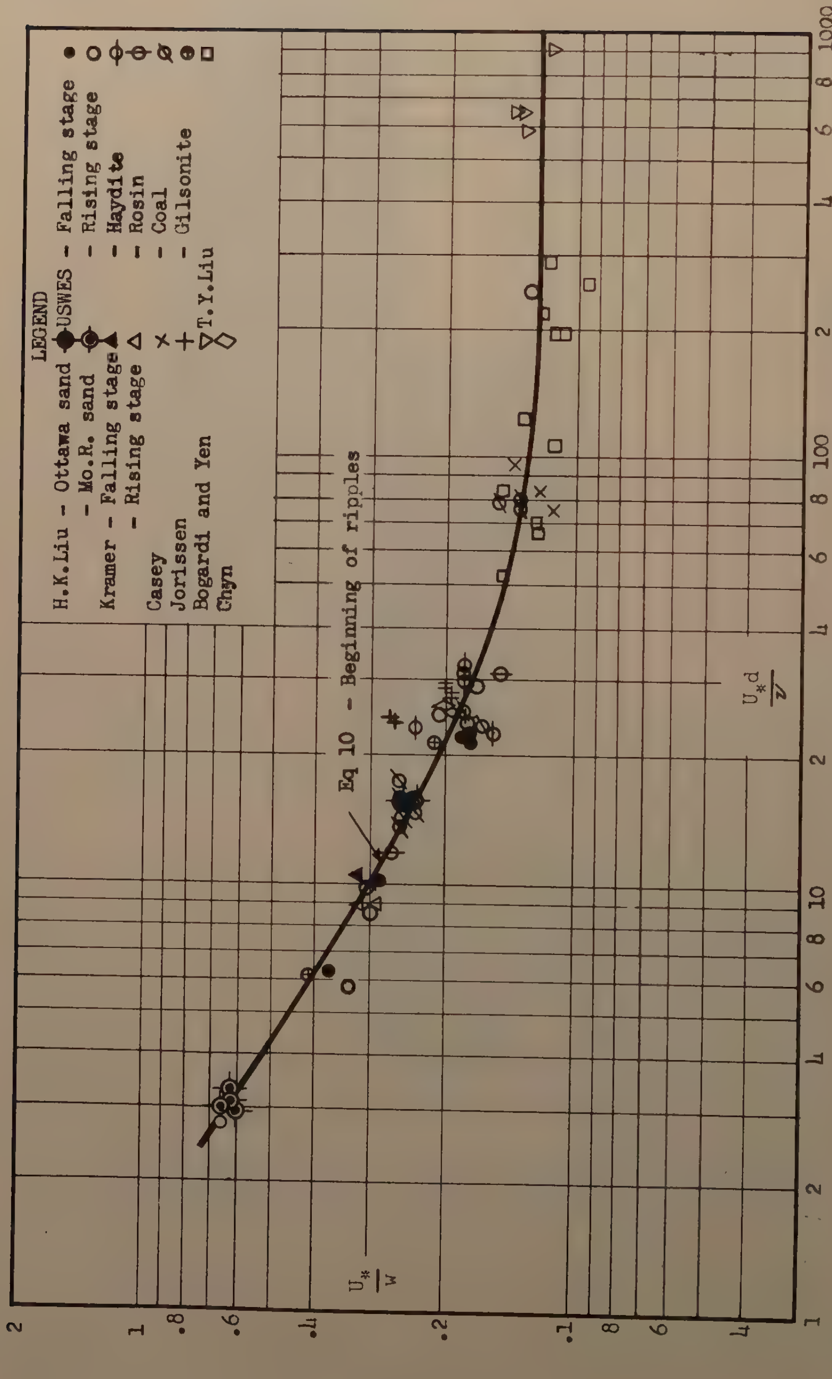

Fig. 9 - Criterion of Ripple Formation as a Function of $\frac{U_*}{w}$ and $\frac{U_* d}{\nu}$.

ripple formation. The scatter of the data may be due to errors in measurement, difficulties of defining the first appearance of ripples, the difference in shape factor of the particles, and the effect of different sediment mixture.

It should be noted that data pertaining to falling stage as well as to rising stage are included. This means that ripples or sand waves will disappear from the bed if the flow conditions fall below the curve in Fig. 9.

The sediment size range is $d = 0.2$ mm to 10.56 mm, the specific gravity range is $S = 1.03$ to 2.70, and the kinematic viscosity range is $\nu = 0.864 \times 10^{-5}$ ft.^{2}/sec. to 1.4×10^{-5} ft.^{2}/sec. Fall velocity of the sediment particles is computed by assuming that the particle is spherical.

It would be interesting to know the relation between the beginning of bed-load movement and the beginning of sediment-ripple formation. There is considerable uncertainty about the criterion of the beginning of bed-load movement. Detailed discussions on this subject are out of place in this paper. In general, the criterion established by Shields(26) has been widely referred to (Curve I in Fig. 11). Shields used the parameter $\frac{T_c}{(\gamma_s - \gamma)d}$ where T_c is the so-called critical tractive force ρU_*^2, γ_s the specific weight of the sediment, and γ the specific weight of the fluid. In order to compare the beginning of bed-load movement with that of sediment-ripple formation, this parameter is transformed into $\frac{U_*}{w}$ by assuming fluid density $\rho = 1.94$ and fluid kinematic viscosity $\nu = 1.2 \times 10^{-5}$ at 60°F. It may be seen that the interval between the two curves is very small (Fig. 10). In fact, for higher Reynolds numbers they actually coincide, which means at the moment the bed load begins to move, the periodic pulsation in the flow is already strong enough to cause sand waves.

Another question with which this research was concerned was whether the parameter $\frac{T}{(\gamma_s = \gamma)d}$ can be used to substitute for $\frac{U_*}{w}$ where T is the tractive force pertaining to the bed at the time of ripple formation; in other words, whether w, can be substituted by its submerged weight. Figure 11, plotted with $\frac{T}{(\gamma_s - \gamma)d}$ vs $\frac{U_* d}{\nu}$, shows considerable scatter, therefore, $\frac{T}{(\gamma_s = \gamma)d}$ is not a satisfactory parameter for the study of sediment-ripple formation. In this study, it is found that $\frac{U_*}{w}$ vs $\frac{U_* d}{\nu}$ is Eq. 10 shown in Fig. 9 constitutes an effective criterion. The question remains whether $\frac{T}{(\gamma_s - \gamma)d}$ is a satisfactory parameter to be used in defining the beginning of bed-load movement. However, this is beyond the scope of this research.

SUMMARY

Through the review of existing literature and comparison of interfacial instability of stratified flow with flow over an alluvial bed, sediment ripples are explained as caused by the instability of the interface, which is a transition layer between the fluid and the bed. Such a transition layer can be in three forms, laminar boundary layer, laminar sub-layer, or a vortex layer.

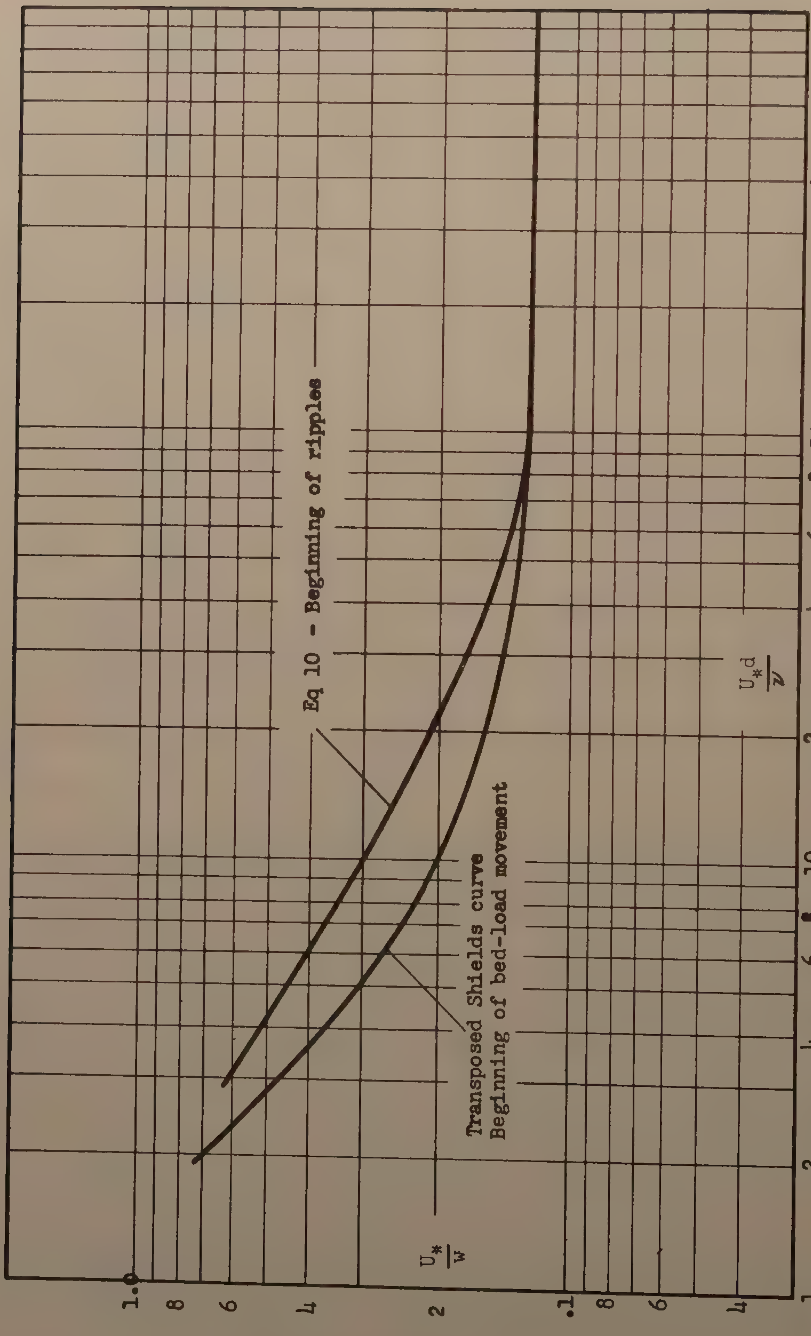

Fig. 10 - Relationship between Beginning of Bed-load Movement and Formation of Ripples

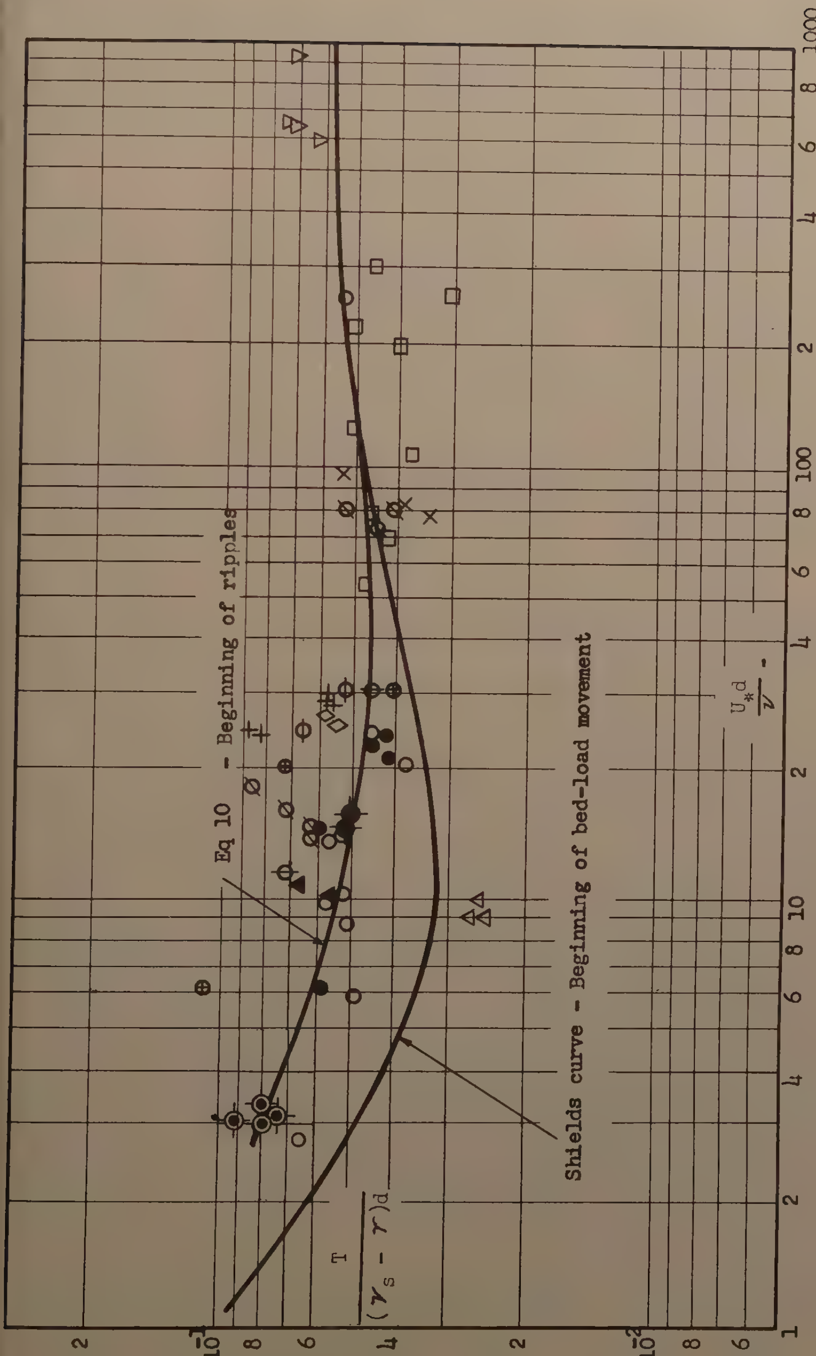

Fig. 11 - Data of Ripple Formation Plotted in terms of $T/(\gamma_s - \gamma)d$ versus $\frac{U_* d}{\nu}$

An index of the interfacial instability is suggested and a criterion of sediment-ripple formation is established in this study. It is shown that other factors such as turbulence, surface waves, and small irregularities of the bed are not the primary causes of sediment-ripple formation although they may affect the movement of sediment ripples.

ACKNOWLEDGMENT

This paper is based mostly upon one part of the author's doctoral dissertation on which Dr. L. G. Straub was the advisor. The experiments were conducted in the St. Anthony Falls Hydraulic Laboratory, University of Minnesota, of which Dr. Straub is director. During the period of research, Dr. A. G. Anderson, Professor of Civil Engineering, University of Minnesota, gave much valuable criticism and advice. Dr. D. P. Peterson, Head of the Civil Engineering Department, and Dr. M. L. Albertson, Head of Fluid Mechanics Research, Civil Engineering Department, both of Colorado A and M College, encouraged the author to publish this research and reviewed the paper. To all who have contributed to this paper the author expresses his gratitude.

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Journal of the
HYDRAULICS DIVISION
Proceedings of the American Society of Civil Engineers

CHARACTERISTICS OF A LARGE THROATED SIPHON

J. C. Stevens,[1] M. ASCE
(Proc. Paper 1198)

SYNOPSIS

A syphon spillway, having what is believed to be the largest throat dimension (9.0 x 35.6 ft.) was designed and piezometrically tested by the author. The paper gives its discharge coefficient, efficiency, primary behavior and other data. It maintains a constant forebay level at the Portland General Electric Company's Sullivan plant regardless of ordinary floods and load variations.

The Sullivan Siphon was built some 4 years ago, following the writer's design, by the Portland General Electric Company during the remodelling and enlargement of its Sullivan Plant at Willamette River Falls 13 miles south of Portland, Oregon.

This siphon is believed to have the largest throat or summit area of any such siphon spillway in this country, perhaps in the world. Its nominal dimensions are 9 ft. high by 35.6 feet long divided into 2 barrels. The siphon was built over an existing wall, between two existing buildings, and supports the plant transformer sub-station on its upper deck.

Tests were made during the winter flood on Willamette River of December 1955, using piezometers that had been built into the north siphon barrel during construction.

The results of these tests, giving the flow, discharge coefficient, efficiency and other data regarding this siphon in action form the subject matter of this paper.

The Sullivan Hydroelectric Plant

This power plant has 13 units for a nominal capacity of 15,000 kw, requiring some 7000 cfs at the lowest operating head of 16 feet, which can occur

Note: Discussion open until September 1, 1957. Paper 1198 is part of the copyrighted Journal of the Hydraulics Division of the American Society of Civil Engineers, Vol. 83, No. HY 2, April, 1957.

1. Cons. Engr., Stevens & Thompson, Portland, Ore.

only during a major flood on Willamette River. The turbine efficiencies at this head are low and any lower head would make it impossible to maintain generation since they are of the induction type.

The tailwater, rises considerably faster than the headwater. However, the headwater is maintained at a constant forebay level of about 58 feet, but it could rise about 5 feet higher without damage to the Crown-Zellerbach paper mill which, with the Sullivan plant and the Government ship locks occupy the left bank of Willamette River at this locality.

Fig. 1 is a plan showing the power plant, siphon, locks, paper mill and Willamette River falls. The river above the power plant has a natural range of 20 feet from 55 feet to a record maximum of 75 feet. The intake gates are normally open at all times. The normal operating level of the forebay is 58 feet (mean sea level) which is also the elevation of the siphon crest. The siphon therefore automatically maintains this forebay level approximately at all times regardless of most floods and load fluctuations. In low water periods, flash floods are put on the crest of the falls to raise the water level in the forebay.

Table I presents data on the river characteristics at this site.

Table I Sullivan Plant Data

Willamette River	River Gage	Tailwater Gage	Forebay Gage	Head on Turbines
cfs				
20 000	55.0 ft	11.0	55	44.0
50 000	58.0	17.0	58	41.0
100 000	61.3	30.0	58	28.0
200 000	66.0	38.0	58	20.0
360 000(2)	71.0	54.0	--	--.-
500 000(3)	75.0	63.0	--	--.-

The Sullivan Siphon

Fig. 2 shows a cross-section of the north barrel of this siphon. The south barrel has approximately the same areas but the dimensions differ somewhat. The only space available for it was that between the paper mill and the power station having an average width of 41 feet and containing a cross wall connecting the two buildings around which the siphon had to be draped. The walls of the two buildings are not parallel.

The elongated crest and sharp curves were necessary to confine the siphon to the available space and to provide room enough for the substation on top.

Four sets of piezometers were built into the north siphon barrel, with small pipes leading to tees, from each piezometer, to the north curb of the

2. The flood of Jan. 8, 1923.
3. Flood of December 4, 1861.

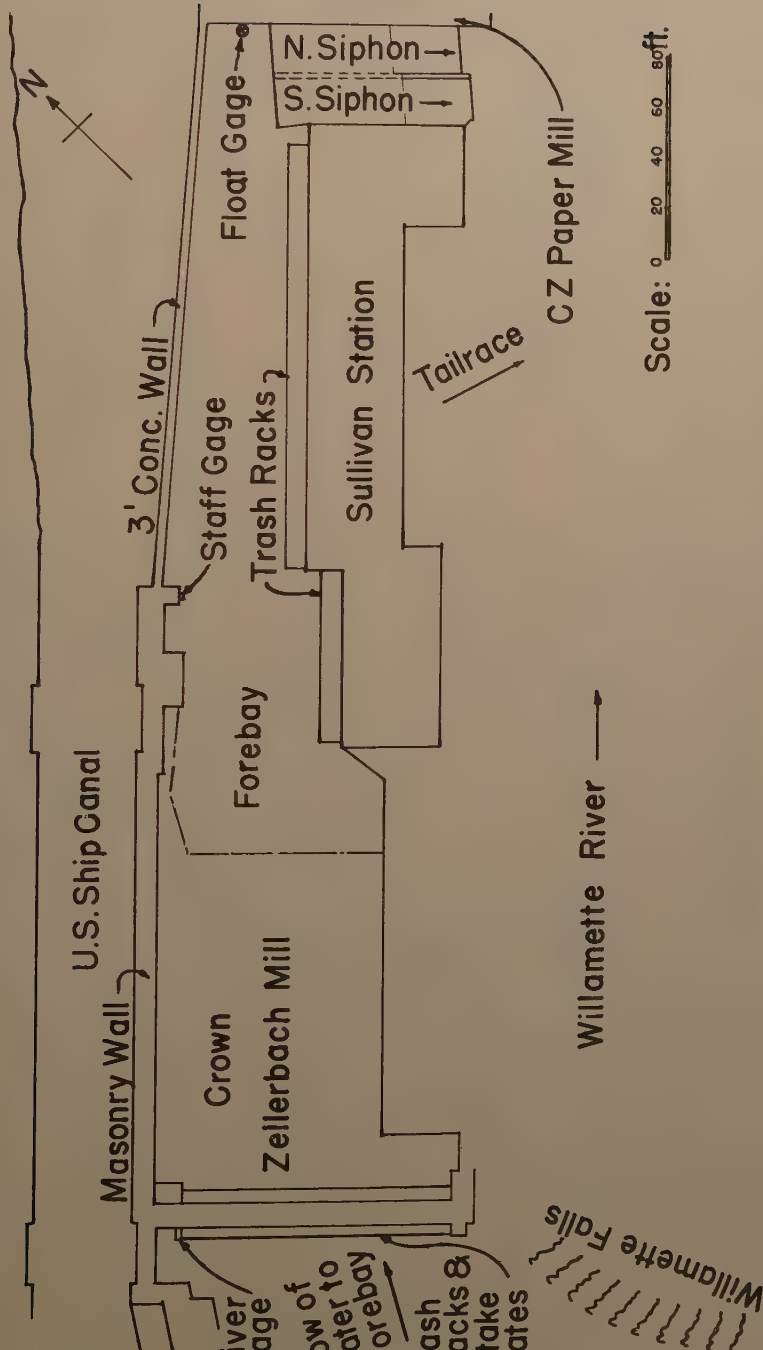

N
N. Siphon
S. Siphon
Float Gage
CZ Paper Mill
Scale: 0 20 40 60 80 ft.
3' Conc. Wall
Staff Gage
Trash Racks
Sullivan Station
Tailrace
Forebay
U.S. Ship Canal
Masonry Wall
Crown Zellerbach Mill
Willamette River
Willamette Falls
River Gage
Flow of Water to Forebay
Trash Racks & Intake Gates

Fig.1

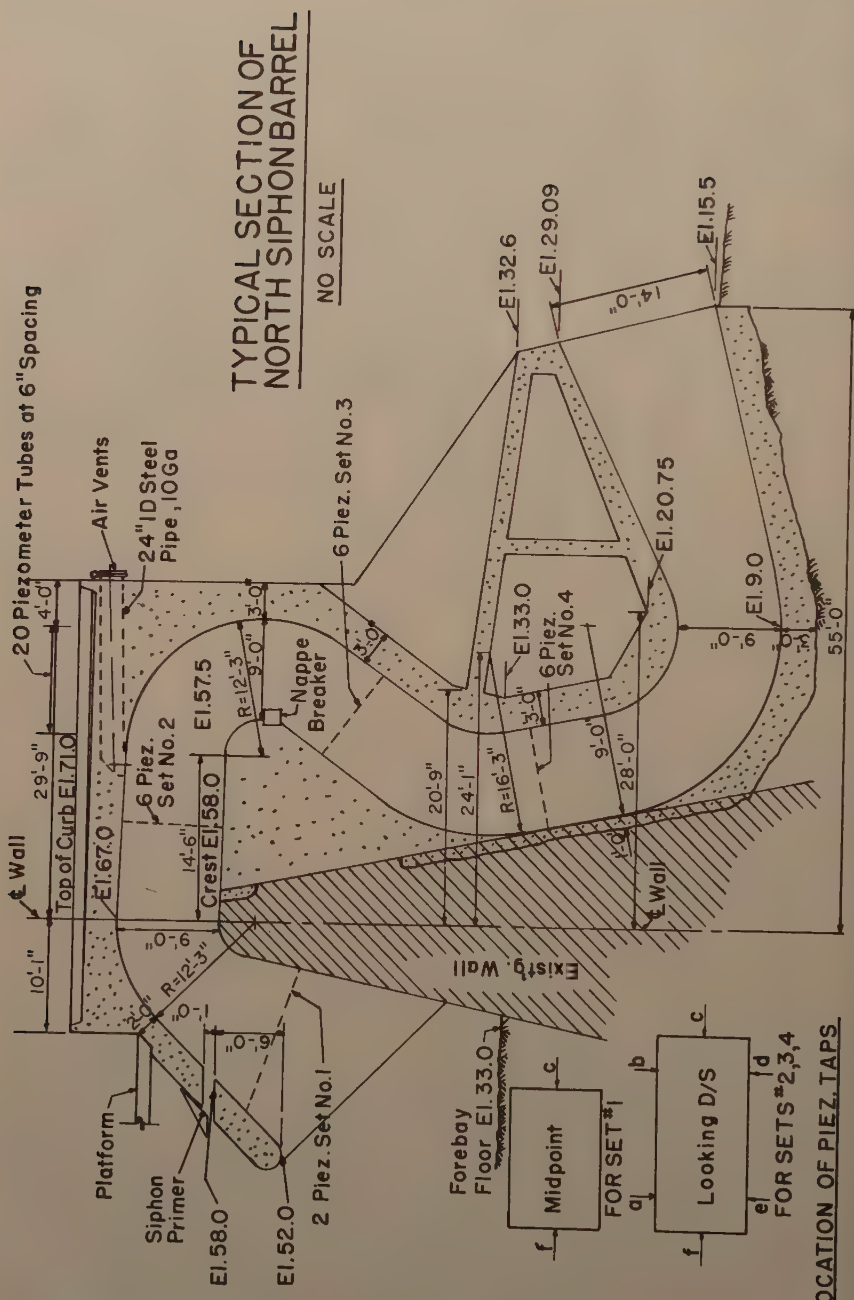

TYPICAL SECTION OF NORTH SIPHON BARREL
NO SCALE
20 Piezometer Tubes at 6" Spacing
Air Vents
24" ID Steel Pipe, 10 Ga
6 Piez. Set No.3
6 Piez. Set No.2
6 Piez. Set No.4
2 Piez. Set No.1
Nappe Breaker
Top of Curb El.71.0
El.67.0
El.57.5
Crest El.58.0
El.32.6
El.29.09
El.15.5
El.20.75
El.33.0
El.9.0
El.58.0
El.52.0
Forebay Floor El.33.0
Exist'g. Wall
℄ Wall
Platform
Siphon Primer
4'-0"
29'-9"
10'-1"
14'-6"
9'-0"
R=12'-3"
3'-0"
14'-0"
20'-9"
24'-1"
R=16'-3"
28'-0"
55'-0"
2'-0"
1'-0"
0'-9"
Midpoint
FOR SET #1
Looking D/S
FOR SETS #2,3,4
LOCATION OF PIEZ. TAPS

Fig.2

Table II Dimensions of the North Siphon Barrel

Section	Distance on ℄ Barrel ft	Dimensions ft	Area sq ft	Elev. of ℄ Barrel ft
Inlet	0	17.5x19.6	343	45.6
Piez Set No. 1	8.6	17.7x13.5	239	52.8
Primer	12.0	17.7x11.0	195	58.5
Piez Set No. 2	31.9	17.8x 9.0	160	62.2
No. 3	60.0	17.9x 9.0	162	46.2
No. 4	78.0	18.1x 9.0	163	29.7
Outlet	114.5	19.0x14.00	266	22.3

deck (Fig. 2) for future tests. Table II gives the essential dimensions of the north barrel.

Each barrel was provided with four 24-inch flap-valve air vents, shown in Figs. 1 and 2, mounted along the down-stream side of the deck. The valve leafs were hinged at the top and inclined just enough to close of their own weight. With the siphon outlet sealed and a rising forebay the air in the barrel is compressed enough to restrict the rise in the forebay and hence delay the priming of the siphon. These air vents release the compressed air, whereupon a vacuum can form within the barrel which pulls the flap valves tightly against their seals.

The upstream end of the summit of the siphon crest is at elevation 58.0. Thus no water flows over the crest when the forebay is at its normal operating level.

The primer consists essentially of (1) a horizontal slot of 1 foot deep in the downward sloping outer face of the siphon, and (2) an adjustable steel plate with its lower face horizontal and the inclined face held by imbedded bolts that pass through slots in the latter face above the angle in the plate. A few vertical steel ribs give stiffness to the primer plate. The adjustment permits the horizontal plate to cover a portion or all of the primer slot. As in Fig. 2 it is set to cover half the opening. The siphon can not begin to prime until the water rises to the under-face of the horizontal portion of the primer plate, thus shutting off the inflow of air to the siphon.

About the center of the curve at the downstream end of the crest is a nappe breaker. This consists of a recess in the concrete that extends the full length of the crest and about 18 inches along the ends of each barrel. This recess permits air to enter from the crest ends during overflow, thus aerating the under-face of the nappe and permitting it to fall free of the invert of the barrel. Thus the overflowing water forms a curtain across the lower leg to carry air out of the siphon.

As above stated the air vent valves, which connect with the interior of the siphon barrels by 24-inch pipes (Fig. 2) are indispensible in starting syphonic action, when quick priming is essential. During the tests first one then an-

other of the flap valves would fly open a few inches, then suddenly close. Some times all the valves would flutter for several seconds just before finally being forced firmly shut by the vacuum forming within.

Model Studies

After the design of the siphon was completed, model studies of it were made at the Hydraulic Laboratory at Oregon State College in the spring of 1952. Obviously without special and expensive facilities the sub-atmospheric pressures within the siphon can not be simulated.

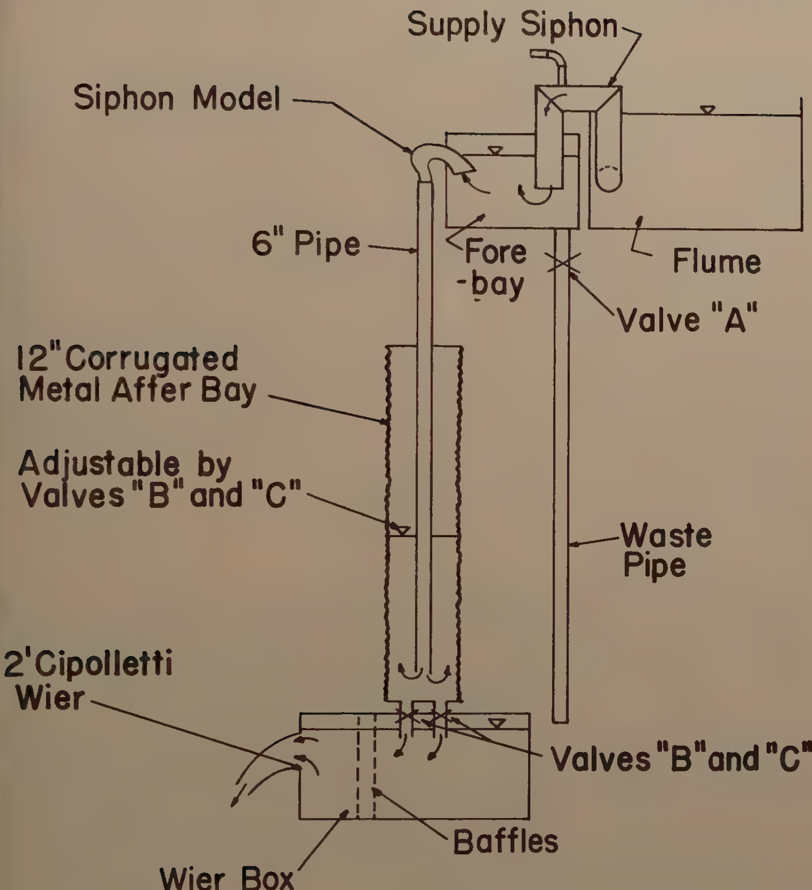
In order therefore to more nearly simulate prototype behavior, the small transparent plastic Laboratory Model was mounted on top of a 28-foot pipe. This, in effect, made a model under a full head by which prototype priming characteristics, vacuums, velocities, air consumption, etc. could approximately be simulated. This model is designated as the High Head Model. It was mounted in a tank built on the side of the PGE company's Sandy River power flume where it crossed a small stream some 35 feet above the creek-bed, some 30 miles east of Portland. The tail pipe discharged into the bottom of a larger corrugated iron pipe with valves in the bottom by which the head on the siphon could be varied. All the flow through the siphon was measured over a 2-foot Cipolletti weir. Fig. 3 is a sketch of the High Head Model.

Table III gives a sampling of the data obtained by the Laboratory and High Head Models. The plexiglass Laboratory Model was not designed for the rough treatment it got when it was put on top of the 28-foot 6-inch tail-pipe. It soon wrecked itself but not before some valuable information was obtained.

Table III - Data from Model Studies

Siphon Head	Vel. for Siphon Head	Flow cfs	Throat Vel.	Outlet Vel.	Discharge coef for Throat Vel.	Discharge coef for Outlet Vel.	Efficiency
Prototype Values from Laboratory Model 1/30 scale							
46.6	58.4	5400	33.4	20.3	.62	.37	.72
37.5	49.1	4950	30.6	18.6	.62	.38	.65
33.0	46.1	4500	28.2	17.0	.61	.37	.61
27.5	42.1	4070	25.4	15.3	.58	.36	.55
23.3	38.7	3800	23.8	14.3	.61	.37	.52
19.3	35.2	3360	21.0	12.7	.60	.36	.46
16.2	32.3	3170	19.7	11.9	.61	.37	.43
15.0	31.1	3000	18.6	11.3	.60	.36	.40
Prototype Values from High Head Model							
26.2	41.0	3380	21.2	12.7	.52	.31	.46
25.5	40.5	3520	22.0	13.2	.54	.33	.48
22.1	37.7	3080	19.3	11.6	.51	.31	.42
21.3	37.0	3240	20.2	12.2	.58	.33	.44
19.0	35.0	2980	18.0	11.2	.51	.32	.39
17.0	33.1	2870	17.6	10.8	.53	.33	.38

The prototype flow for the High Head Model can be found from the model flows since the velocities are approximately the same as for the prototype.



HIGH HEAD MODEL INSTALLATION

Not to Scale

Fig. 3

They are slightly less due to friction in the tail pipe. Since the Laboratory Model was to a 1/30 scale the High Model flow is 900 times the Model flow.

Automatic Forebay Level Regulation

An important feature of the design of this type of siphon is that when the inflow to it diminishes the syphonic action is not broken but the siphon con tinues to discharge on part rations as it were. During such time the siphon flows partly full and rhythmically gulps air as the inflow diminishes. The High Head Model behaved in this manner and the Sullivan siphon itself followed that model to a marked degree as had other siphons designed by the author. The Laboratory Model however did not depict this characteristic.

The Walterville[4] siphons will run indefinitely on any flow between 1 and 100 percent of their maximum capacity. The Leaburg siphons[5] would not break their siphonic action until the flow fell below about 10% of their maximums. It is believed that the Sullivan siphon will also continue to discharge indefinitely at least 10 percent, perhaps less, of its maximum without losing its priming, although no direct test of this could be made.

The design of the siphon intake and the priming features have much to do with its partial-flow-air-inhaling qualities, which provide such siphons with most excellent forebay level regulating properties.

Tests of the Siphon

The first test was made shortly after the remodeled Sullivan plant was put into operation. It was made to determine whether the siphons would really function. The piezometers were not used. The plant was carrying a full load. All units were shut down simultaneously. The forebay surged to a 2.7 - foot rise, both siphon barrels primed, the test was over and the company officials were quite satisfied.

The author however still had a curiosity to see how such a big throated siphon behaved inside. Therefore under a promise of cooperation a manometer bank of 20 U-tubes, 36 inches in height, was constructed on a plywood panel during the fall of 1954 for the author by Leupold & Stevens Instruments Inc. in anticipation of the time when piezometer tests would be made.

Piezometric Tests of December 23, 1955

After all preparations were made for tests in 1954 no flood of sufficient magnitude occurred to justify them and it was not until the general high water of December 1955 that such tests appeared possible. In preparation therefore the manometers were half filled with mercury, hauled to the site and connected to the 20 piezometer taps shown in Fig. 2.

The services of Prof. Roy Shoemaker of Oregon State College, who had worked on the Siphon Tests, were enlisted to assist and make the photographs of the manometer bank. Splendid cooperation was had with the engineering staff of the Portland General Electric Co. and especially the Sullivan plant

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4. Siphons as Water Level Regulators by J. C. Stevens, Trans. ASCE Vol. 104 (1939) p. 1787.
5. On the Behavior of Siphons by J. C. Stevens, Trans. ASCE Vol. 99 (1934) p. 986.

superintendent and his assistants. December 23 was a cold, raw, rainy day but the river was near the peak of flood, and in spite of the approaching Christmas holiday the crew remained on duty until the tests were completed.

Aside from the manometer bank a float gage had been installed in the forebay. One end of a steel tape was attached to a float in a stilling well. The other end carried a counterweight while the tape ran over a pulley. Readings to the nearest 100th foot, mostly at 5-second intervals, were made while the tests were under way.

The plan followed in each test was as follows: With the power plant running steadily prior to the tests a number of units were suddenly shut down. Shortly after more units were shut down. The water thus rejected from the generating units went into forebay storage until the forebay level raised enough to prime the siphons. During the tests photographs were taken of the manometer bank as appeared expedient.

Knowing the approximate turbine efficiencies, the load on each unit at the time of shut-down and the change in forebay levels it became possible to calculate the siphon discharge with sufficient accuracy for all practical purposes. In fact there is no other possible way of determining the discharge through this siphon.

NOTATION

(Foot-second units)

Let H = elevation of the forebay (sea level datum)
- t, time in seconds from shut down of the units
- T, flow through the turbines
- S, storage rate in cfs during the time, t
- h, head of overflowing water on the siphon crest
- q, free flow over the south siphon barrel crest
- Q, flow through the north siphon barrel
- I, inflow to the forebay from the river
- R, river level just above the intake gates
- O, outflow from forebay except that through the north barrel
- A, cross-sectional area
- α, energy coefficient of mean velocity head

The horizontal cross-sectional area of the forebay was practically constant at 30,000 sq. ft.

The average storage rate therefore was

$$S = 30{,}000 \frac{\Delta H}{t} \tag{1}$$

The head on the siphon crests was

$$h = H - 58.0 \tag{2}$$

The free flow over either barrel crest was

$$q = 46.0\, h^{3/2} \tag{3}$$

During the tests the river gage above the intake to the forebay (Fig. 1) stood at 62.0 and the tailwater gage averaged 40.0 feet above sea level.

The inflow to the forebay was equal to the flow through the turbines when the forebay gage showed 58.0 ft. or less. When the forebay level increased free flow occurred over the crests of both siphon barrels until the water reached 58.5 whereupon the siphon air intakes were automatically sealed and the process of priming began.

Test No. 1

This test was made primarily to test the method and equipment. Both siphon barrels were in operation. No complete analysis of the results was made because the siphons although primed were running only about half full.

The forebay, prior to the test, stood at 58.6 and the air intake to both siphon barrels was shut off at the primers. The units running in the plant just prior to this test with the load and estimated flows were as follows:

Units	kw	T-cfs
1, 2, 3 & 9	1500	2100
4, 5, 6 & 8	1600	2220
10, 11, 12 & 13	1200	1680
	4300	6000

Unit No. 7 was down for repairs.

It appears from this that the total initial inflow to the forebay was 6000 cfs. The schedule of unit-shut-downs was as follows:

Time pm	Units off	cfs	T-cfs	Manometer Photos Taken		Forebay gage
12/23/56			6000	No. 8 at	2:35	58.6
2:33	12&13	840	5160	No. 6	2:39	58.6
2:38	10&11	840	4320	No. 5	2:42	58.7
2:39	6&8	1110	3210	No. 3	2:44	58.8
2:41	4&5	1110	2100	No. 4	2:45	58.8
		3900				

But little water went into storage during this test so the two siphons could take only about half their combined capacity. An axamination of the piezometer readings indicated there was a depth of about 5 feet of water flowing over the north barrel crest. Moreover, both siphons just before the loads on the units were restored at the conclusions of the tests, were inhaling substantial quantities of air.

With the aid of the air vents both siphons primed shortly after the 1st two units were shut down on a head of 0.6 ft. on their crests.

Test No. 2

For the second test on this date the south siphon barrel was prevented from priming by tying open 3 of the 24-inch air vents. Water therefore flowed freely over the south barrel crest.

The following data obtained for the test:

River gage just above the forebay intake gates	62.0 ft.
Forebay gage just prior to beginning the test	58.3
Tail water gage	40.0
Head on the turbines at beginning of test	18.3

The schedule of unit-shut-downs was as follows:

Time	Units off	cfs	T-cfs	Manometer Photo
12/23/56			6000	No. 1 at 3:26
3:27	10,11,12&13	1680	4320	No. 7 at 3:30
3:31	6&8	1110	3210	No. 7A 3:33
3:35	4&5	1110	2100	No. 8A 3:36

The governors maintained a constant load on each unit running, as shown by the graphic kw meter for the plant, so that even though the forebay gage reached a maximum of 60.1 feet during the test, the flow through the turbines did not vary appreciably.

As the forebay level increased water went into storage but was withdrawn again after the siphon primed. The inflow to the forebay decreased as the forebay level increased because of the reduction in head on the intake gates from river to forebay, while the gate openings remained constant.

The flow through the turbines was based on an over-all efficiency of 46%. The normal head on the turbines is 40 feet or more, for which the overall efficiencies are fairly high but at only 18 feet head the water used increases disproportionately. The relation between kilowatts load and water usage for a head of 18.3 feet then becomes

$$T - cfs = 1.40 \text{ kw} \tag{4}$$

The inflow to the forebay is made up of several factors (see notation)

thus

$$I = T + S + q + Q \tag{5}$$

from which the flow through the north barrel of the siphon becomes

$$Q = I - (T + S + q) \text{ or } Q = I - 0 \tag{6}$$

The outflow, except that through the north siphon barrel, is given by the items in ( ) above, which for convenience includes storage in the forebay.

Table IV shows how the flow through the siphon was calculated.

Table IV Calculated Flow Through North Siphon Barrel during Test No. 2 Dec. 23, 1956

Time h M s (1)	H (2)	ΔH (3)	S cfs (4)	R-H (5)	$\sqrt{\frac{R-H}{3.7}}$ (6)	h (7)	$h^{3/2}$ (8)	q (9)	T (10)	O (11)	I (12)	Q (13)
3:26:00	58.30			3.7	1.00	.30	.16	10	6000	0	6000	10
3:29:00	60.10	1.8	450	1.9	.72	2.10	3.04	140	4350	4940	4300	(140)
3:30:00	59.95	-.03	-90	2.05	.75	1.95	2.72	120	4350	4380	4500	-120
10	59.86	-.09	-270	2.14	.76	1.86	2.54	120	4350	4200	4600	400
20	59.70	-.16	-480	2.30	.79	1.70	2.22	100	4350	3970	4700	770
30	59.50	-.20	-.600	2.50	.82	1.50	1.84	80	4350	3830	4900	1070
40	59.30	-.20	-.600	2.70	.85	1.50	1.34	80	4350	3830	5100	1270
50	59.03	-.27	- 800	2.97	.90	1.03	1.04	50	4350	3600	5400	1800
3:31:00	58.80	-.23	- 700	3.20	.93	.80	.72	30	4350	3680	5600	1920
10	58.68	-.12	- 360	3.32	.95	.68	.56	30	3230	2900	5700	2800
20	58.60	-.08	- 240	3.40	.96	.60	.46	20	3230	3010	5750	2140
30	58.60	0	0	3.40	.96	.60	.46	20	3230	3250	5750	2500
40	58.65	+05	+150	3.35	.96	.65	.52	20	3230	3400	5750	2350
50	58.70	05	150	3.30	.95	.70	.59	30	3230	3410	5700	2290
3:32:00	58.78	08	240	3.22	.94	.78	.69	30	3230	3500	5600	2100
3:33:00	58.80	02	10	3.20	.93	.80	.72	30	3230	3270	5600	2330
3:34:00	58.80	0	0	3.20	.93	.80	.72	30	3230	3260	5600	2340
3:35:00	58.80	0	0	3.20	.93	.80	.72	30	3230	3260	5600	2340
3:35:30	58.90	.10	100	3.10	.92	.90	.85	40	2100	2240	5500	3200
3:39:45	58.90	.05	10	3.05	.91	.95	.93	40	2100	2150	5500	3350
3:40:00	58.90	0	0	3.05	.91	.95	.93	40	2100	2140	5500	3360
:30	58.90	0	0	3.05	.91	.95	.93	40	2100	2140	5500	3360
3:41:00	59.00	.05	50	3.00	.90	1.00	1.00	50	2100	2200	5400	3200
:30	59.00	0	0	3.00	.90	1.00	1.00	50	2100	2150	5400	3250
3:42:00	59.00	0	0	3.00	.90	1.00	1.00	50	2100	2150	5400	3250

Cols. (5) & (6)

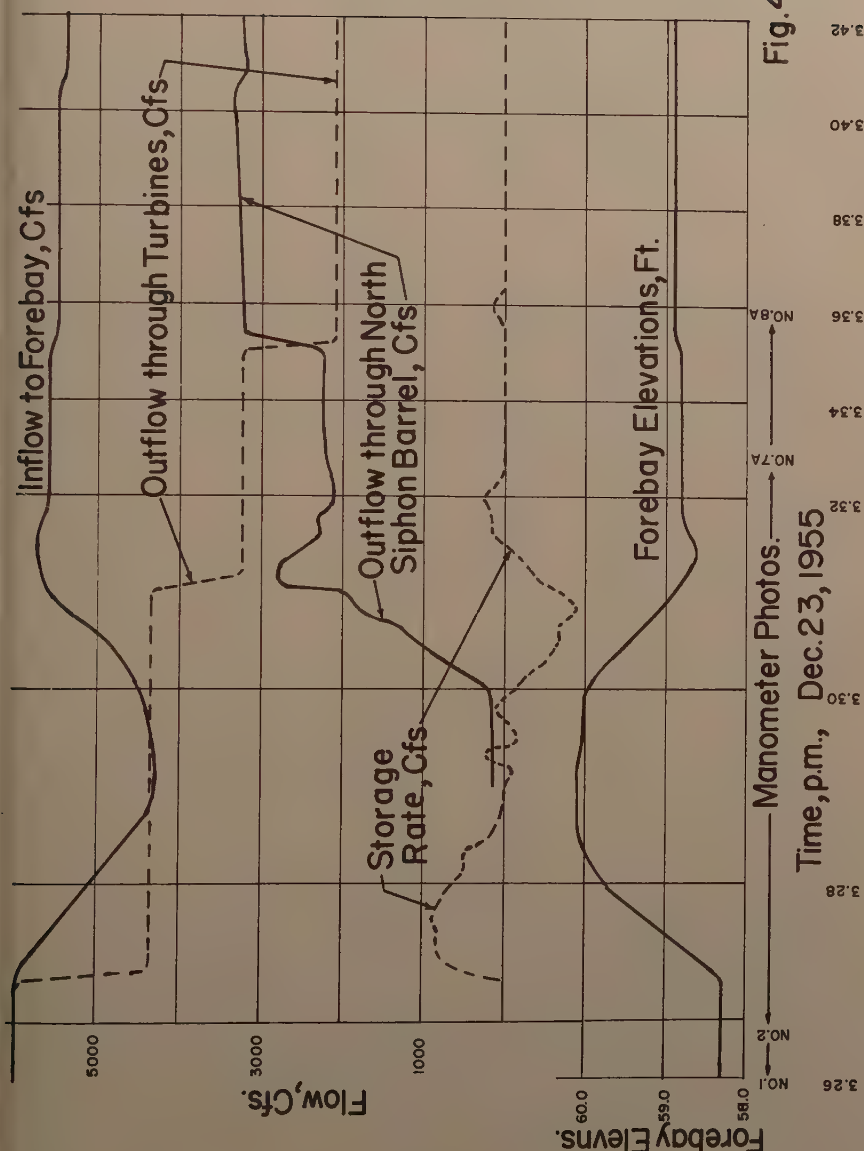
The head on the intake gates is R-H. As the forebay builds up this head diminishes and the inflow is reduced by the factors in Col. (6). This is the only thing that affected the inflow since the river and the gates were stationary. Col. (12) is obtained by multiplying 6000 (the initial inflow) by the factors in Col. (6).

Col. (13)

The second item of 140 cfs. is the free flow over the north barrel crest. Application of the formula Q = I-O would result in a negative flow of 640 cfs. obviously impossible. The siphon had not primed and surges were oscillating through the forebay. The calculations for siphon flow were not begun until th surges quieted and the siphon began to prime.

Fig. 4 shows graphs of the essential data in Table IV which are self-explanatory.

Table IV shows a maximum flow of 3400 in round figures. No photographs of the manometers were taken in that interval 3:40:00 to 3:40:30. The head o the siphon was then 18.9 ft. Correcting this flow to a head of 16 feet, a flow c 3100 cfs would result. This is 11% less than the originally calculated siphon



Inflow to Forebay, Cfs
Outflow through Turbines, Cfs
Outflow through North Siphon Barrel, Cfs
Storage Rate, Cfs
Forebay Elevations, Ft.
Flow, Cfs.
5000
3000
1000
Forebay Elevns.
60.0
59.0
58.0
NO.1
NO.2
NO.7A
NO.8A
Manometer Photos.
Time, p.m., Dec. 23, 1955
3.26
3.28
3.30
3.32
3.34
3.36
3.38
3.40
3.42

Fig. 4

Table V Data from Manometer Photo No. 8A

3:36 pm Dec. 23, 1955, Siphon flow 3200 cfs

Piez-ometer	Manometer inches mercury	Manometer Feet Water	Elev of Piez	Pressure Line Water	Pressure Line Absolute
1 f	0	0	50.5	50.5	83.5
1 c	.8	.9	55.1	56.0	89.0
Av.	+.4	.5	52.8	53.3	90.5
2a	-8.1	-9.2	66.7	57.5	90.5
2b	-9.7	-11.0	66.7	55.7	88.7
2c	-7.6	-8.6	66.2	53.6	86.6
2d	-3.6	-4.1	57.7	53.6	86.6
2e	-3.4	-3.9	57.7	53.8	86.8
2f	-7.6	-8.6	62.2	53.6	86.6
Av.	-6.7	-7.6	66.2	54.6	87.6
3a	-4.2	-4.8	47.6	42.8	75.8
3b	-3.3	-3.7	43.7	40.0	73.0
3c	-12.8	-14.5	42.6	28.1	61.1
3d	-14.8	-16.8	43.7	26.9	59.9
3e	-14.0	-15.8	47.6	31.7	64.7
3f	-11.1	-12.6	50.9	38.3	71.3
Av.	-10.0	-13.3	46.2	34.6	67.6
4a	-1.4	-1.6	29.0	27.4	60.4
4b	0	0	30.5	30.5	63.5
4c	0	0	30.6	30.6	63.6
4d	+0.8	+0.9	30.5	31.4	64.4
4e	+0.6	-0.7	29.0	28.7	61.7
4f	-0.1	-0.1	28.6	28.5	61.5
Av	0	0	29.7	29.7	62.6
Siphon inlet (forebay)				58.9	91.9
outlet (tailwater)				40.0	73.0

flow of 3500 cfs under the minimum operating head of 16 feet. The reason for this is found in the low flow coefficient due to the sharp curves that had to be adopted at each end of the long crest and the angle of inflow in order to fit the siphon into the available space. It is not possible to estimate flow in a projected siphon under such circumstances in advance much closer.

Manometer Data

The analyses of the photographs made possible the determination of the characteristics of the siphon. Only the data from Photo 8A taken at 3:36 pm while the siphon was carrying 3200 cfs are presented herein. The locations of the piezometers are shown in Figs. 2 and 5. Data from Photo 8A are given in Table V.

Characteristics of the Sullivan Siphon

At the time Photo 8A was taken of the manometer bank at 3:36 pm the flow through the north siphon barrel as seen in Table IV was 3200 cfs, the forebay gage showed 58.9, and the tailwater gage marked elevation 40.0.

From these data it is possible to determine the flow coefficient, the efficiency, and the pressure and energy gradients throughout the length of the siphon. These data are shown in Table V (see also Table II).

Table VI Siphon Characteristics

Distance ℄ Barrel	Elev of ℄	Area sq ft	Mean Velocity	$V_m^2/2g$	Water Head	α	$\alpha \frac{V_m^2}{2g}$	Absolute Energy Head	Station
0	45.6	343	9.3	1.07	58.9	3.5	3.8	95.7	Inlet
8.6	52.8	239	14.4	3.22	53.3	2.75	8.9	94.7	Piez Set No. 1
31.9	62.2	160	20.0	6.22	54.6	1.0	6.2	93.8	Piez Set No. 2
6.0	46.2	162	19.6	5.97	34.6	3.0	18.0	85.6	Piez Set No. 3
78.0	29.7	163	19.6	5.97	29.7	3.0	18.0	80.7	Piez Set No. 4
114.5	22.3	266	12.0	2.24	40.0	2.0	4.5	77.5	Outlet

The data in Table V are shown in Fig. 5 based on the profile of the center line of the siphon drawn by plotting the elevation of the center line against its distance from the inlet. The absolute head of course is the water head plus 33.0 ft. (one atmosphere).

If we add the velocity head of the mean velocity to the absolute pressure we obtain the dotted line marked "Pseudo Energy Head" which cannot correctly represent the energy line of the siphon since an energy line must always slope in the direction of flow because energy is being continually absorbed by friction, curves, eddies, etc. The only way the correct delineation of the energy gradient can be brought about is by the use of an energy coefficient.[6]

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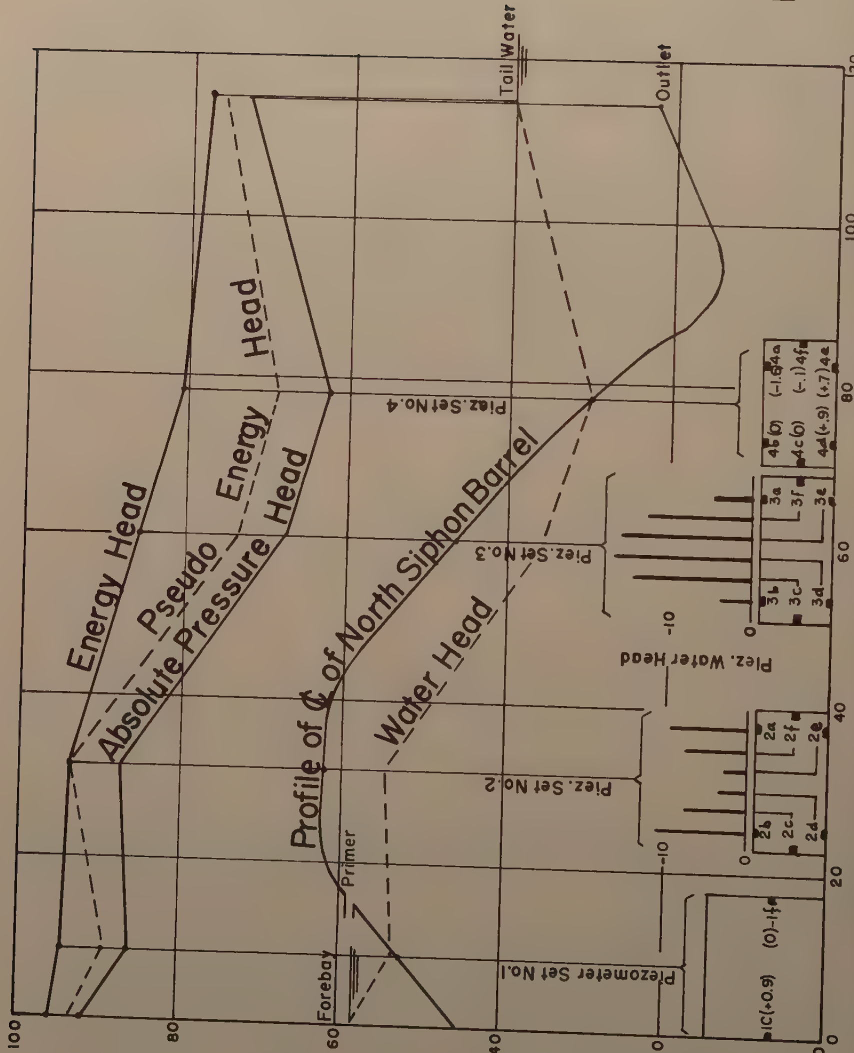
6. Velocity Head correction for Hydraulic Flow, by Morrow P. O'Brien and Joe W. Johnson, Engineering News-Record, August 16, 1934, p. 214.



Energy Head
Pseudo Energy Head
Absolute Pressure Head
Profile of ℄ of North Siphon Barrel
Water Head
Tail Water
Outlet
Forebay
Primer
Piezometer Set No.1
Piez. Set No.2
Piez. Set No.3
Piez. Set No.4
Piez. Water Head
1C(+0.9)
(0)-1f
2a
2b
2c
2d
2e
2f
3a
3b
3c
3d
3e
3f
4b(0)
4c(0)
4d(+.9)
(-1.6)4a
(-.1)4f
(+.7)4e
0
-10
0
20
40
60
80
100
120

Fig.5

This coefficient is given by

$$\alpha = \frac{\int_0^A V^3 dA}{V_m^3 A} \tag{7}$$

Where V is the mean velocity in the sub-areas into which the cross-sectional area is divided. A is the entire cross-sectional area, and V_m is the mean velocity of the entire area = Q/A.

This coefficient can only be determined by painstaking velocity traverses throughout the area in question which is seldom possible. In cases such as this we are, however, fully justified in giving values to α that will give the energy gradient a definite slope downward, i.e., take out the anomalies in the pseudo energy line. In justification for the values given this coefficient in Table V the following facts may be cited: (1) a glance at Fig. 1 will show that the water flowing into the siphon must make a sharp right angle turn from the forebay. The effect of this is seen in Fig. 5 at Piezometer Set No. 1 (looking downstream) where the water head at 1 c on the outside was .9 foot greater than 1 f on the inside of the section. This effect was damped out to some extent at Set No. 2 since the heads at 2c and 2f are practically equal; (2) The entrance curvature and the vertical curves combine to cause a very complex pattern of excessive turbulence throughout the siphon barrels. At the vertical curves the flowing water is mostly on the outside while violent eddies occupy the inside portions of the curves. The mean velocity is nowhere equal to Q/A even though a coefficient of α = 1.0 was used at Set No. 2. To have done otherwise would have required larger values of the coefficient α elsewhere to no good purpose.

Coefficient of Flow[4] is defined as the ratio of the mean velocity at the outlet to the velocity corresponding to the siphon head. In this case we may use the maximum calculated discharge of 3400 cfs in round figures, the mean velocity of which at the outlet is 3400/266 = 12.8 fps and the velocity corresponding to the siphon head of 18.9 ft. is 34.9 fps, therefore the flow coefficient becomes

$$C = 12.8/34.9 = .37 \tag{8}$$

This compares favorable with the coefficients obtained by model tests, Table III. Both the flow coefficient and the efficiency are low on account of the sharp entrance skew, and curves the water encounters in traversing the siphon barrel. Those of the High Head Model are less than that of (8) because of friction in the tail pipe.

Efficiency[5]

This characteristic of a siphon is not the same as its flow coefficient. It is defined as the ratio of the actual summit velocity to that corresponding to one atmosphere, i.e. the ratio of the actual to the maximum passible discharge through the throat section. Such a condition would obtain in any siphon

4. Ibid, p. 1799; also reference (5) p. 1002 which contains an extended discussion of this coefficient.

when a head of one atmosphere is used in providing the entrance and friction losses and producing the summit velocity. The summit velocity for this test is given by $\frac{3400}{160}$ = 21.2 fps. The velocity for 1 atmosphere, 33 feet, is 46.1, hence

$$E = \frac{21.2}{46.1} = .46 \tag{9}$$

Priming

As shown under Test No. 1 both barrels primed on a head of 0.6 ft. This was on a slowly rising forebay (2 units at a time shut down at an average of 2-minute intervals). For the fast rising and surging forebay of Test No. 2 it took a head of 1.8 ft. over the crest before the north barrel primed on the flow from the 4 units shut down at the beginning of the test. It took 2 1/2 minutes for the forebay to stabilize, and the siphon to begin priming.

The priming time of all siphons depends largely on the rate of rise of the forebay and on whether the siphon is provided with air vents. The priming of the Laboratory Model showed little or no relation to that of the prototype. That model would not prime at all unless the outlet was sealed by the tail-water and also unless its air vent was functioning. With the air vent closed it would not prime at all even if the forebay flooded over the top of the model.

The High Head Model on the other hand, with the air vent in operation, primed under all conditions of tail-water, even without any, due doubtless to the aspirating effect of the 28-foot column of falling water. With the air vent closed, however, this model could be over-topped by a rising forebay, as well as the Laboratory Model, without its priming.

CONCLUSIONS

A laboratory small-scale model of a proposed siphon can determine the approximate flow that the siphon will carry, the discharge coefficient and the efficiency. In Table III the flows given for the 1/30 scale model at 19 and 16 feet head compare favorably with observed values of its prototype. The flow coefficients based on both models are reasonably close to prototype values. However, the priming and air inhaling characteristics are no dependable index of prototype behavior.

The High Head Model has some promising possibilities for future research. If used the forebay and tail-water levels must be determined with considerable accuracy and they must be stable.

In designing siphons an endeavor should be made to avoid sharp curves and skew in the approach channels if high coefficients and efficiencies are desired. Expanding the outlet will reduce the flow coefficient based on the outlet velocity but will increase the efficiency, and reducing it will have the opposite effect.

It is difficult to guess how high the summit throat of a siphon section may be and still assure proper priming qualities. The suction height of pumps and the draft height of turbines have been limited, through experience, to about 25 feet. It may be possible therefore to design a siphon with a summit barrel 20 to 25 feet in height, if conditions should make such necessary, and expect the siphon to function properly. Just what the priming qualities of such a siphon might be is conjectural. A model siphon to about a 1/10 scale under a full

head, such as was used herein, would furnish valuable data on the characteristics of such a high-throated siphon or any other type, provided it is sturdy and the water level controls will insure stable conditions during tests.

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OCEAN WAVE FORCES ON CIRCULAR CYLINDRICAL PILES

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(Proc. Paper 1199)

SYNOPSIS

The forces exerted by ocean waves on circular cylindrical piles were studied at an exposed location near Davenport, California, in water from forty-five to fifty feet in depth. These studies were made by the University of California under contract to the Signal Oil & Gas Company. Measurements were made of forces resulting from waves as high as twenty feet by means of a strain gage type force meter. These measured forces are presented in graphical form. The pile test sections were 6-5/8 inches, 12-3/4 inches, two feet and five feet in diameter. Coefficients of drag and mass were computed from the measured data using linear wave theory to predict the water particle velocities and accelerations. The coefficient of drag was found to have no well-defined relationship to Reynolds number in the test range of Reynolds number from 3×10^4 to 9×10^5. The average value of the coefficient of mass was found to be 2.5 with a normal Gaussian distribution about this average value. Within the limits of accuracy of the tests, and within the range of conditions, there appeared to be no relationship between the coefficient of mass and the flow conditions as represented by the Reynolds number, with water particle acceleration, or with wave period. Large lateral vibrations of the 2 foot pile were noted under the action of high waves, probably due to the alternate breaking off the large vorticies which were noticeable. The period of these vibrations was approximately 2.5 seconds, with the wave period being about 13 seconds. As a result of the effect of this vibration force, combined with the effect of the continual alternating forces due to the drag and inertia forces, the six inch central supporting pile broke in the middle seat of the threads at a distance of 3 feet below the bottom collar. An examination of the break indicated that it was a fatigue failure.

Note: Discussion open until September 1, 1957. Paper 1199 is part of the copyrighted Journal of the Hydraulics Division of the American Society of Civil Engineers, Vol. 83, No. HY 2, April, 1957.

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INTRODUCTION

For several years the Wave Research Laboratory, University of California, Berkeley, has been engaged in research on the problem of forces exerted by water gravity waves on piles. A theory has been developed to account for both drag and inertia forces.[17,18] The applicability of the theory has been tested in the laboratory and a few tests have been performed in the field.[1,20,21,22] In order for the theory to be useful to engineers in the design of offshore structures, it was necessary to undertake a comprehensive field test for the purpose of determining the coefficients of drag and mass which appear in the equations. In addition, it was necessary to obtain data at a series of submergence depths of the test sections in order to determine the distribution of forces with respect to distance below the surface. The study described herein was conducted near Davenport, California, at a location exposed to large waves where measurements were made of forces exerted by waves as high as twenty feet in water which varied between about forty-five and fifty feet in depth. The wave forces were measured by strain gages mounted on restraining bars, the outer ends of which were attached to the test section and the inner ends of which were attached to a supporting central pile. Measurements were made of forces exerted on a 12-3/4 inch and a 2 foot diameter test pile section. The equipment was modified, and measurements were made of forces exerted on a 6-5/8 inch test section. The equipment was again modified, and a few measurements were made of forces exerted on a 5 foot section; however, the supporting unit failed before adequate data could be obtained.

Theoretical Considerations

In the design of a pile structure exposed to waves of a given height and period, some of the factors involved are the size, shape, and spacing of the piles, and the distribution of moments on the various members. Theoretical and experimental investigations have shown that the force exerted on the sections consists of two parts, a drag force and an inertia force. Two methods have been developed to handle the problem. Although the particle motion in waves is orbital, the methods are based upon rectilinear flow. The first method, described by Morison, O'Brien, Johnson and Schaaf,[17] is based upon the assumption that the drag force and the inertia force can be treated separately and that the total force can be obtained by adding the solutions linearly. This is admittedly an assumption which is open to question. However, until a greater insight is had of the flow phenomenon, this is a useful tool for the design engineer. The equation contains two coefficients which have to be determined empirically.

In steady rectilinear flow, the drag force has a magnitude which depends upon Reynolds number. At very low Reynolds number, the drag force consists predominantly of shearing forces over the surface of the pile; however, when the turbulence of the fluid and the geometry of the system are such that a turbulent boundary layer exists and separation occurs, the drag force is proportional to the fluid density, the projected area and the square of the fluid particle velocity, and is usually represented by the equation

$$F_D = C_D \rho A u^2 \; * \tag{1}$$

* See List of Symbols for definition of symbols.

The inertia force is proportional to the fluid density, the volume of the object and the fluid particle acceleration. The inertia force is commonly represented by the equation

$$F_M = (M_o + M_v)\,\frac{\partial u}{\partial t} \tag{2a}$$

This may be written as

$$F_M = C_M\,\rho\,V\,\frac{\partial u}{\partial t} \tag{2b}$$

The total horizontal force exerted on a vertical differential section, dS, along the pile is[17]

$$dF = \left[\tfrac{1}{2}\,C_D\,\rho\,D\,|u|\,u + C_M\,\rho\,(\pi D^2/4)\,\frac{\partial u}{\partial t}\right] dS \tag{3}$$

Where $|u|u$ has been introduced in place of u^2 to maintain correct sign convention.

The equations for the undisturbed horizontal components of water particle velocity and acceleration as obtained from the linear theory of water gravity waves are[19]

$$u = \frac{\pi H}{T}\,\frac{\cosh\,(2\pi S/L)}{\sinh\,2\pi d/L}\,\cos\,2\pi\,t/T \tag{4}$$

$$\frac{\partial u}{\partial t} = \frac{2\pi^2 H}{T^2}\,\frac{\cosh\,(2\pi S/L)}{\sinh\,(2\pi d/L)}\,\sin\,2\pi\,t/T \tag{5}$$

These are only good first approximations for small values of H/L for values of d/L greater than about 0.2.[12,19] There are more complex solutions, which have a greater number of terms. Although these solutions are useful in some respects, laboratory studies indicate that they do not predict the velocities or accelerations with a greater degree of accuracy than do the simple equations presented above.[19] It is believed that the engineer might be best served by using these simple formulas and then using an adequate safety factor in his design. The hyperbolic functions used in these equations may be obtained from published tables.[23]

In the linear theory the maximum drag and inertia forces are 90° out of phase and the resultant maximum force on the pile leads the crest of the wave (or the trough, when the force is in the opposite direction). The relative importance of the inertia force increases with the ratio of pile diameter to wave height. The limiting effect occurs when the diameter is so great that the structure begins to act as a section of a sea wall; theoretically the equation is valid only for infinitesimal values of D/L. Details on obtaining the total force on a pile will not be considered herein as they have been given in Reference 17.

The second method, described by Crooke,[6] is based upon the study by Iversen and Balent[15] of the forces exerted upon a body in accelerated motion through a fluid. This method assumed that there was a linear dependence of velocity upon acceleration so that the force could be expressed as the product of one coefficient, the fluid density, the projected area of the body, and the square of the particle velocity.

The equation for the wave force on a pile segment is:

$$dF = \tfrac{1}{2}\,C\,\rho\,D\,|u|\,u\;\;dS \tag{6}$$

where u is given by Equation 4. In the work of Iversen and Balent[15] it was found that the total resistance coefficient, C, was related primarily to Iversen's Modulus which expresses the product of the acceleration and the body diameter divided by the square of the velocity and also to the Reynolds number.

For the case of wave action on piles, it is evident that the past history of the water particles plays an important part in the explanation of the action by either of the methods described above, in addition to the factors which have been mentioned.

Experimental Equipment

Preliminary field studies of wave forces on piles were made by measuring the force necessary to restrain a pile suspended by a hinge, using strain gages mounted on restraining bars as the transducer. This system permitted the measurement of the total movement about the hinge; however, it did not permit the direct determination of force distribution. Further, the dynamics of the system was such that the entire pile vibrated with a period which was nearly that of the locally generated wind waves: this served to invalidate many of the data. Because of this, a force meter[3] was developed to allow the measurement of the force exerted by waves over a small section of pile, and to decrease the effect of vibrations.

Force Meters

In general, the system consisted of a section one foot in length of the particular diameter pile that was to be tested mounted around a six-inch (6-5/8" O.D. pipe) central support pile. On the inside of this section—the "test section"—mounting shoes were fastened. On the outside of the central pile restraining bar clamps were fastened. The test section was connected to the central support pile by bolting restraining bars to the mounting shoes and to ears on the restraining bar clamps (Figure 1). One bar was mounted on each side of the pile. Thus, even though the central support pile was subject to forced vibrations, the force being measured was due to the relative motion between the test section and the central support pile; hence, only a relatively small vibration would appear on the force record, except under extreme conditions. The actual test section was designed so that its natural frequency was very high (order of magnitude 15 to 50 cycles per second, depending upon restraining bar) compared with the wave period. For complete design details the reader is referred to Reference 3.

The above condition applied only so long as the amplitude of pile vibration was small enough so that the forces induced by the movement of the test pile back and forth through the water were negligible. In practice, this was found to be the case for small submergence depths of the test section. For greater submergence depths it became necessary to guy the bottom of the central support pile to prevent excessive motion.

In order to have natural flow conditions about the test section, a shroud of the same diameter as the test section was mounted above and below the test section. The two shrouds were each five feet in length (Figure 2).

The transducer consisted of two Baldwin Southwark SR-4 paper backed strain gages mounted on each of the two restraining bars between the center and the top of the bar. The strain gages were mounted as close as possible

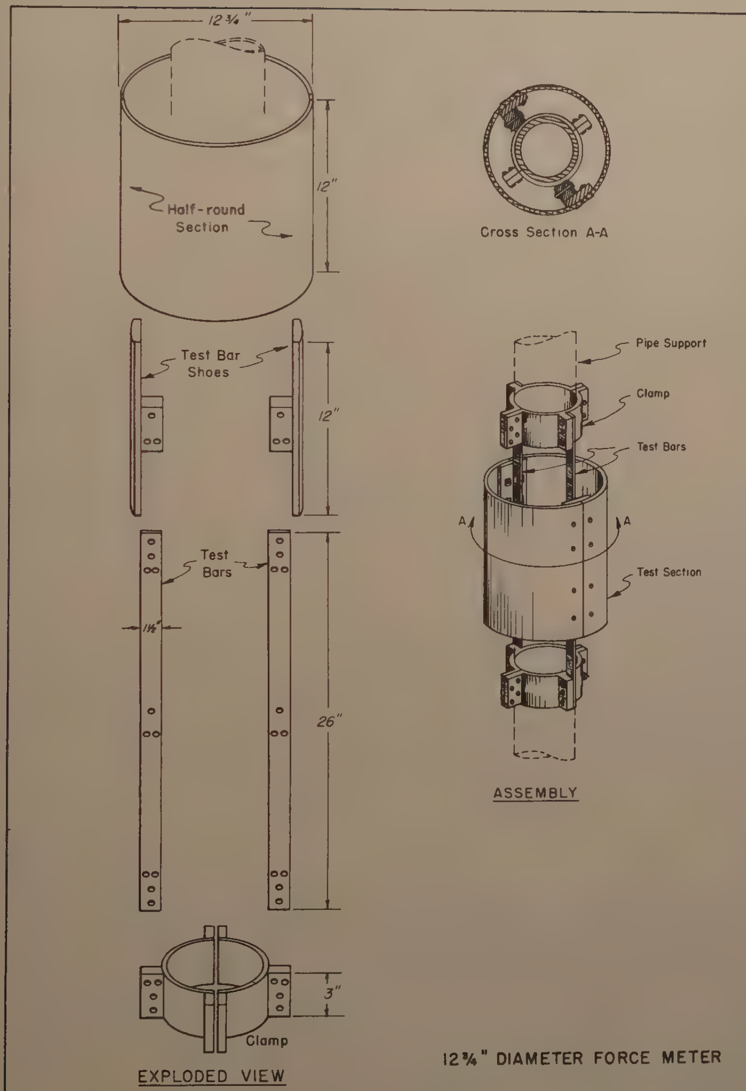

12 3/4"
12"
Half-round Section
Test Bar Shoes
12"
Test Bars
1 1/2"
26"
3"
Clamp
EXPLODED VIEW
Cross Section A-A
Pipe Support
Clamp
Test Bars
A
A
Test Section
ASSEMBLY
12 3/4" DIAMETER FORCE METER

FIGURE 1

to the points of maximum test bar moment for maximum sensitivity. The strain gages were glued to the flat side of the restraining bar by a special glue and then waterproofed by means of Okonite tape, a primer and neoprene. The waterproofing was found to be excellent, although considerable care had to be taken during the many steps to avoid the development of "pinholes" in the neoprene. Bars submerged at sea for several months remained in a satisfactory condition. Several sets of strain bars were manufactured, with different sensitivities, so that measurements could be made for a wide range of forces.

Tirex cable was used to connect the strain gages to the bridge of a Brush Universal Analyzer, which, in turn actuated the Penmotor Oscillograph. The connections were waterproofed with Okonite cement and tape.

Although the test pile system was subject to large bending moments, calculations showed that the error induced in the force measuring system by pile deflection due to such moments would not exceed four-tenths of a per cent at most, and that for approximately ninety-five per cent of the data the induced error did not exceed one-tenth of a per cent. A comparison of the moments of inertia of the test bar with that of the drill casing indicated that the drill casing second moment was approximately 10,000 times that of the test bars. Since the moment taken by each component of a system in bending was proportional to the moment of inertia of the component in question, it could be seen that only a small error would be induced into the strain measured by the strain gages.(5)

Pile Installation

The sections of the pile were installed using a fifteen foot high tripod derrick, made of 2-1/2 inch standard pipe legs approximately 6 feet apart at the base and made rigid by use of turnbuckles mounted midway up the legs, with a two-ton chain hoist suspended from the crown (Figure 2).

For the case of the 12-3/4 inch and 2 foot diameter test piles, the bottom joint, twelve feet long, was installed and lowered until the top was just above the upper pile bracket, at which stage it was secured by means of a dog-collar slip clamp. A six-foot joint was then placed on the chain joist, lifted and fitted into the top of the bottom joint and the joints securely made up using two sets of six-foot chain tongs. The pile was extended to the desired length by use of a series of six-foot joints. When the bottom joint had reached the lower work platform, the pile was secured by "U" bolts at the lower pile bracket, the test section and the shrouds mounted, and the test section (force meter) calibrated. The test section and shrouds were painted with anti-fouling paint and the pile was then lowered to the desired depth of submergence, oriented so that the restraining bars were facing the waves, and clamped top and bottom in position (Figure 3).

Minor modifications were made in installing the 6-5/8 inch and the 5 foot diameter test piles.

Calibration of Force Meter

The wave force meter was calibrated using a device built around a four-inch channel about four feet long, the ends of which were fastened to the shrouds above and below the test section by use of metal straps and "U" bolts. A hole was loçated in the center of the channel, and the channel mounted so that this hole lay opposite the center of the test section. Through this hole a

link was placed which was connected to one end of a proof ring and to a strap and "U" bolt around the test section (Figure 5). The other end of the proof ring was fastened by means of a small "J" bolt to a support which held the proof ring in a vertical position. The proof ring contained a Federal C-21 Dial Indicator which had been calibrated prior to the field installation. The calibration of the force meter was performed by tightening the nut on the "J" bolt, which pulled on the proof ring, which in turn pulled on the test section. The readings on the dial were noted on the record on the Bush penmotor recording the strain gage deformations.

Wave Recorders and Stadia Boards

Two Beach Erosion Board Step Resistance Gages of the parallel type were used to measure the wave heights.(4) They were mounted on either side of the test pile, at a distance of approximately twenty feet from the piles, and were oriented in such a manner that the line formed by the gages and the test pile faced directly into the prevailing swell (Figures 3 and 4). The recorders were calibrated by immersing each of the four five-foot sections which made up the gages in the water separately and noting this on the recorder. It was found that the calibration was approximately linear. The outputs from the wave gages were recorded on Esterline-Angus millimeters. In addition, the seaward gage was wired in parallel to one channel of the Brush penmotor so that the waves were recorded on the same chart as were the force records; thus, there was no problem of matching records from two charts. Sola transformers were used to maintain constant voltage to the wave recorders.

In addition to using the wave recorders, a record of the waves was obtained from the motion pictures taken during tests by the installation of two stadia boards in the field of view of the camera (Figure 2) or by painting the test pile in alternate light and dark bands and using it in place of stadia boards (Figure 3). The latter system was found to be satisfactory even under the most adverse conditions. The wave direction was determined by direct observation.

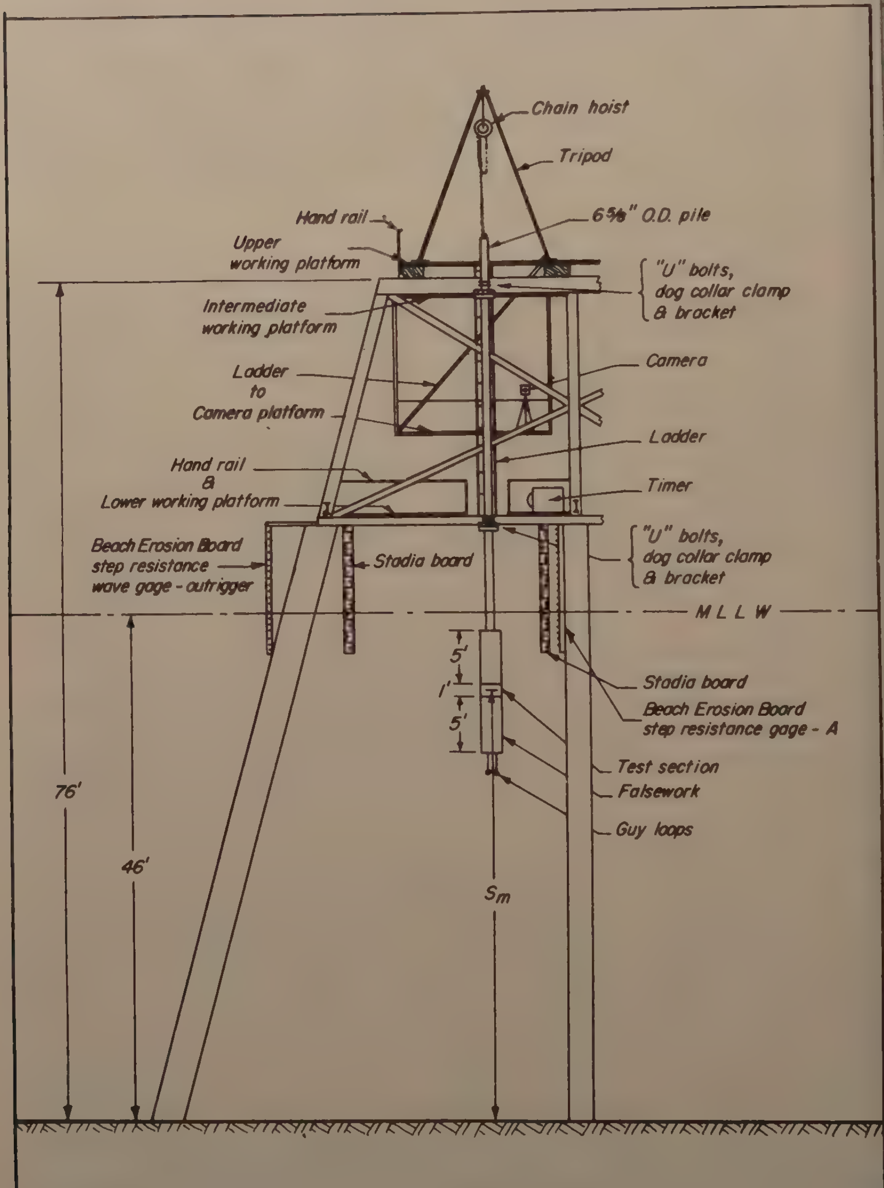
Photographic Equipment

Both 35 mm and 16 mm motion pictures were taken during tests in order to have an independent source of wave measurements and phase angles (Figure 6). These movies were taken from a camera platform which was hung from the main pier deck about twenty feet landward of the test pile. The data on the movie film were correlated with the data on the Brush penmotor by locating a special timer in the field of view of the camera.

Timing Equipment

A special duodecimal timing unit was used during the tests. The system was made visual by means of lights, the pattern of which showed the time to the nearest second, and a rotating black and white disk which showed the fraction of the second to the nearest one-tenth second (Figure 6) and from which even more refined time estimates could be made. In addition, a camera-penmotor synchronization light remained on during each test run.

The timing unit gave a time history of the test. The flashing lights showed on the film, as did the rotating disk, while chronograph pens which were synchronized with the lights made "pips" on the left hand side of the



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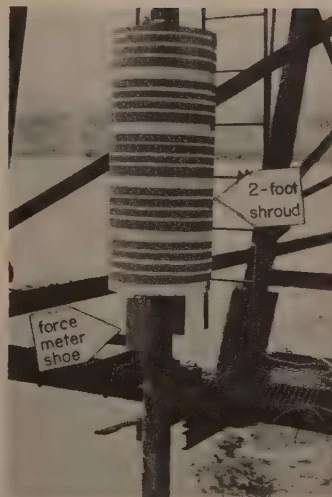
SCHEMATIC DIAGRAM OF TEST EQUIPMENT

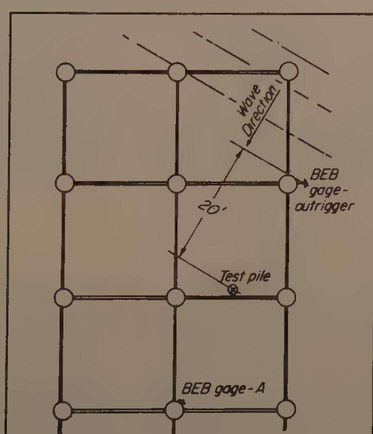
FIGURE

a. 2-foot test pile in wave trough



b. 2-foot test pile partially assembled.

FIGURE 3



Plan view of test section of pier

FIGURE 4

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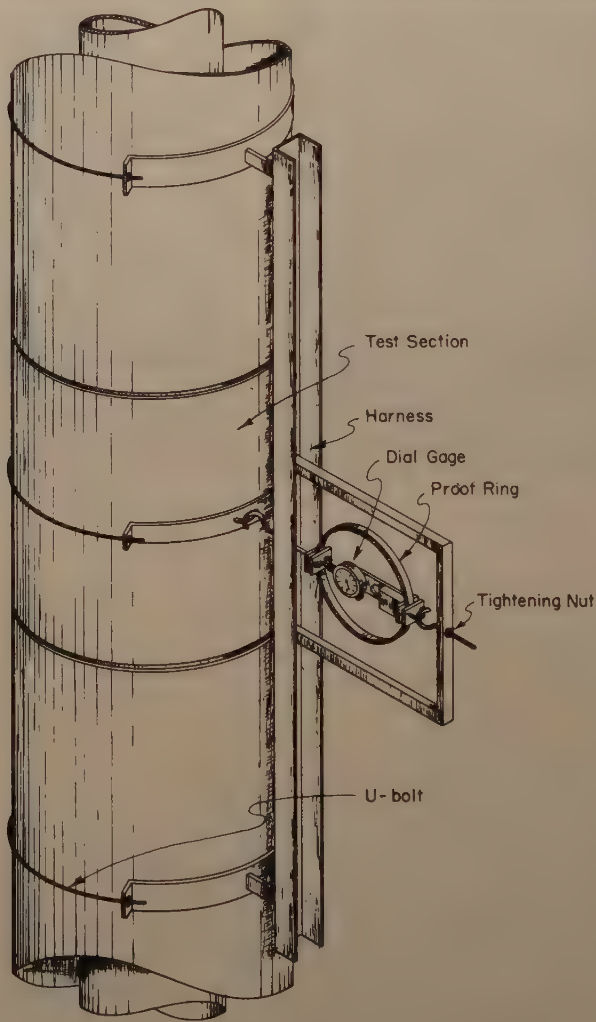

METER CALIBRATION

FIGURE 5

Enlargements from a section of 35mm movie film showing a 16-foot wave passing the $12\frac{3}{4}$" pile and work platform; note timer.

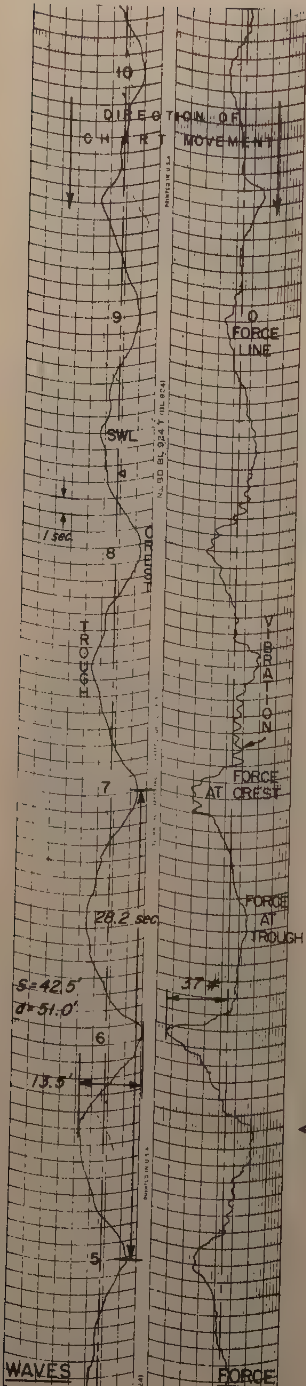

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FIGURE 7a
12 3/4-INCH TEST PILE
Data from Roll 24A
21 December 1953
Davenport, Calif.

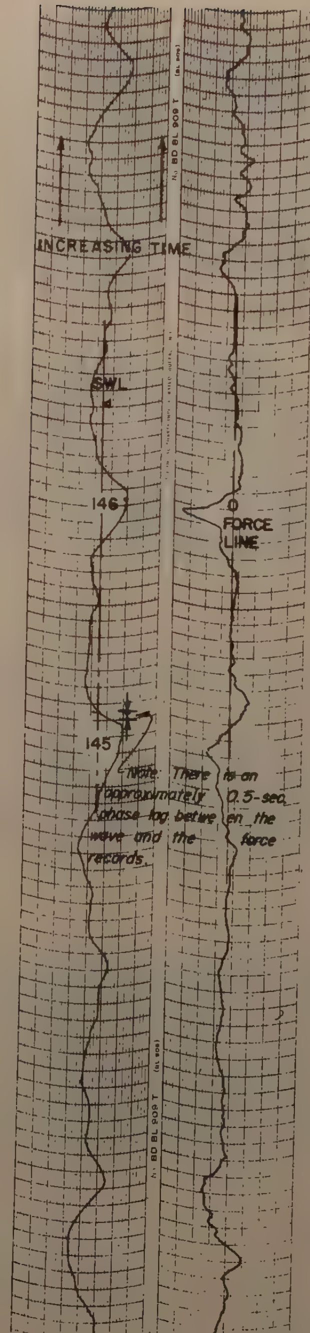


FIGURE 7b
2-FOOT TEST PILE
Data from Roll 51
13 February 1954

FIGURE 7

Brush penmotor chart every second. Every ten seconds a pip was made on the right hand side for ease in determining the time position.

The timer lights had relays which would arc when actuated. This arcing was broadcast, and the signals picked up by the Brush Analyzer, making pips every second on the strain gage side (right hand side) of the penmotor. Initially these interferred with the wave force trace, but the trouble was eliminated by installing an isolation transformer between the timer and the Brush Analyzer.

Power

As the equipment was all installed at the sea end of the pier, approximately twenty-two hundred feet from the shore, the problem of getting adequate power to the end of the pier was critical. It was found after the first tests that the voltage dropped to as low as 90 volts, and the Brush Analyzer would not work properly at this low voltage. Consequently, 440 volt transformers were installed at the shore end and the sea end to decrease the power loss. No trouble was encountered thereafter.

Experimental Procedure

After the pile had been installed at the desired depth of submergence, the equipment was checked out by means of a complete test run. The several "bugs" that were picked up have been described under the various sections under "Experimental Equipment," and the remedies discussed. The equipment was then left in proper operating condition pending the arrival of large waves. The various units were calibrated from time to time during this interval. Some measurements were also made and the results checked with previous field and model data, and found to be satisfactory.

The first major storm occurred during November, 1953. At this time the field crew checked the orientation of the test pile, set up a 35 mm motion picture camera on the camera platform, started the Brush Analyzer so that it could warm up, installed the timer in the field of view of the camera, and then started all the recording equipment. The force was recorded on one channel of the Brush penmotor, and the surface-time history at the seaward BEB gage on the other channel. In addition, the surface-time history of the water surface at the pile was recorded photographically so that two independent sources were available to measure the variables of wave height and wave period.

In order to prevent the non-operation of certain units in case of a power overload, truck batteries and inverters were maintained on a standby basis to be used to run the Brush Analyzer and the 35 mm motion picture camera. The system was found to be satisfactory and with slight modifications was used for the rest of the tests.

Analysis of Data

Wave Data

The data obtained by use of the Beach Erosion Board step-resistance gages were analyzed wave by wave for height and period. As can be seen in Figure 7, for the one-foot test pile the wave crest was nearly at the same position as the maximum force. This was because the BEB gage was located

21 feet seaward of the pile and the waves present at that time took approximately 0.5 seconds to travel from the gage to the pile. Hence, for the purpose of measuring phase relationships, it was necessary to shift the wave record by the equivalent of 0.5 seconds.

The gravity water waves of the ocean are non-uniform, yet the theory for wave forces on piles assumes uniform waves. One is thus faced with the problem of how to apply the theory for uniform waves to the case of non-uniform waves. It is possible, so long as the linear theory holds, to utilize a Fourier integral and the appropriate kernel in a manner similar to that used by Fuchs(12) to predict certain linear effects from instrument records of wave motion, or to make a Fourier analysis of the wave system and utilize the individual components. These methods are tedious to apply and unless their use leads to remarkable improvements from a design standpoint over the more simple method, they will probably not find general application.

The simple method used for the data presented herein was to assume each wave to be approximated by a train of waves of the same amplitude and period as the measured wave. If only the more distinct series of waves are used, as was the case, this method is reliable. The period was measured directly on the chart, and was considered to be one-half the time interval for two successive waves to pass the gage. For the example shown in Figure 7A, which is reduced in size from the original record, the period of Wave 6 was taken to be one-half the time interval necessary for the crest of Waves 5 and 7 to pass (1/2 x 28.2 = 14.1 seconds). The vertical distance between the crest and the preceeding trough was used as the wave height. The values read off the Brush recorder were transformed from chart divisions to feet by means of the proper calibration graph. For Wave 6, Figure 7A, the wave height was found to be 13.5 feet.

Force Data

A sample of the forces exerted on the force meter, as recorded on the Brush Strain Analyzer, is shown in Figure 7A. The maximum force was taken as the vertical displacement of the crest or trough on the force record from the line of zero-force. The reading of the chart was then transformed into pounds of force by use of the appropriate calibration curve. For example, the maximum force in the direction was 37 pounds for Wave 6.

It is readily apparent that the force was not symmetrical about the line of zero force. This was because the data were obtained in relatively shallow water. The waves were not symmetrical; the horizontal components of velocity and acceleration in the direction of movement of the wave crest were greater than the components in the opposite direction. Because of this, the maximum horizontal component of force acting in the direction of wave motion was greater than the maximum horizontal component of force acting in the opposite direction. Vibration type force traces (Figure 7a) were noticeable when a series of nearly periodic uniform waves passed the test pile. During the portions of the records where the waves were irregular the vibration type traces were practically non-existent.

Coefficients of Drag and Mass

For the case of a force exerted on a section of pile, the vertical dimension of which is small compared with water depth and wave length, it can be

shown that the force equation can be approximated quite accurately by:(13)

$$F = \frac{dF}{dS} \Delta S, \text{ or } \frac{dF}{dS} = \frac{F}{\Delta S} \tag{7}$$

where

$$\frac{dF}{dS} = \tfrac{1}{2} C_D \rho D |u_c| u_c + \frac{\pi}{4} C_M \rho D^2 \left(\frac{\partial u}{\partial t}\right)_c \tag{8}$$

and

$$u_c = \frac{\pi H}{T} \frac{\cosh (2\pi S_c/L)}{\sinh (2\pi d/L)} \cos 2\pi t/T \tag{4}$$

$$\left(\frac{\partial u}{\partial t}\right)_c = \frac{2\pi^2 H}{T^2} \frac{\cosh (2\pi S_c/L)}{\sinh (2\pi d/L)} \sin 2\pi t/T \tag{5}$$

where the subscript $-_c$ refers to conditions at the center of the pile test section.

The values of ΔS, d, and S_c were known for each test, and the values of F, H, and T were measured on the Brush recorder. It was thus possible to solve for C_D when $\cos 2\pi t/T = \pm 1$ (when the crest or trough of the wave passed the pile); and it was possible to solve for C_M by measuring the force record when $\sin 2\pi t/T = \pm 1$ (when the crest or trough of the wave passed the pile); and it was possible to solve for C_M by measuring the force record when $\sin 2\pi t/T = \pm 1$ (when the wave passed through the still water level for linear wave theory).

In addition, the Reynolds number was calculated, using the pile diameter and the maximum horizontal particle velocity at the test section.

Resistance Coefficient

The resistance coefficient in Equation 6 was determined for values of the Iversen modulus

$$\frac{\frac{\partial u}{\partial t} D}{|u| u} \tag{9}$$

for various values of $2\pi t/T$. Because of time limitations on the contract this method was used for only a few waves.

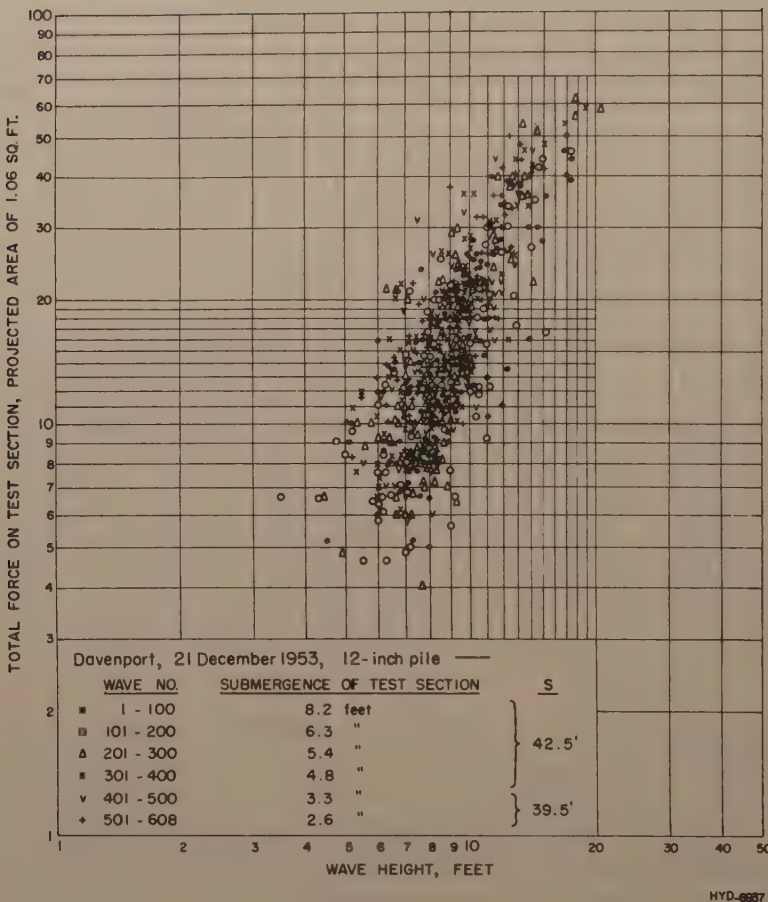
Results and Discussion

Force on Pile Sections

The basic data obtained during these tests are illustrated in Figures 8 and 9. These figures show the maximum force exerted on the one-foot and the two-foot test sections where both sections are one foot high. The scatter of data is not excessive when one considers the variation of periods, the large difference in wave shape from the theoretical, the varying degree of turbulence in the flow, especially the eddies, and the roughness of the piles.

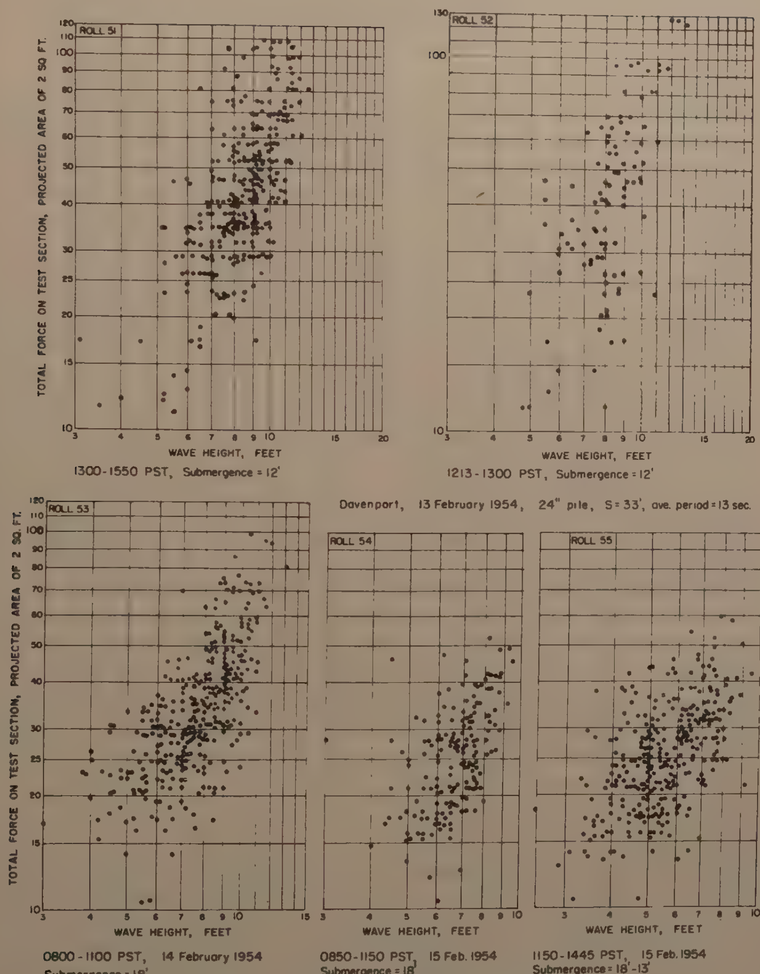
Distribution of Force with Distance Below Still-Water Level

In Figure 10 data are presented the distribution of force with distance below the still-water surface. These data were obtained using the two-foot test



WAVE FORCE AS A FUNCTION OF WAVE HEIGHT, 12$\frac{3}{4}$" Dia. Pile

FIGURE 8



WAVE FORCE AS A FUNCTION OF WAVE HEIGHT (2-foot Pile)

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FIGURE 9

section and locating it at different submergence depths, and measuring waves for a long period of time at each submergence. The data were plotted as wave height versus force for each submergence and then the average values for heights of 4, 5, 6, 7, 8, and 9 feet were picked off the graphs and plotted.

Impact Forces

One of the most interesting parts of the curves shown in Figure 10, from a design standpoint, are the data taken two feet below the still-water level. As the waves were all 4 feet, or higher, the test section was almost always exposed in the wave trough. As the crests of the waves approached, therefore, the test section was resubmerged, but without noticeable impact.

This lack of impact is not surprising, although of considerable importance, when one considers the studies of Bagnold(2) and Denny.(8) They found that even in the laboratory conditions had to be controlled very carefully to obtain impact because it was necessary for the waves to curl over in such a manner as to trap a certain shaped volume of air between the wave front and the face of the structure. They found that impact did not occur if there were even slight variations in conditions. It is thus necessary for a wave to be in the process of curling over in breaking in order to have impact on a cylindrical pile. This is a very rare phenomenon except in the breaker zone. In deeper water the breaking encountered is usually that of the wind blowing the top off a wave and causing a "whitecap."

Both Bagnold and Denny commented that the occurrence of impact force was accompanied by a rather loud sharp noise. This was noticed at Davenport during the presence of large waves. The cross bracings of the pier consisted of H and L beams; the lowest horizontal members were H beams, with the web horizontal. As the large waves traveled the length of the pier a booming noise was heard as the crest passed each horizontal member. It is believed that as the crest passed and completely submerged the lowest bracings, the vertical component of water motion trapped air in the bottom half of the H beams, and that the dimensions and dynamics were such that impact occurred.

Lateral Vibration of Test Section

With large waves, approximately 12-feet high and of 13-second period, as obtained by averaging the highest one-third of the total number considered, it was noticed that considerable lateral vibration of about a 2-1/2-second period occurred for the two-foot diameter test pile. The pile was guyed from the bottom by two wires at an angle in the neighborhood of thirty degrees with the direction of the prevailing waves. In all subsequent tests three wires were used. Part of the vibration may be explained by the two guy-wire arrangement; however, it is believed that the major contribution came from the periodic pealing off of very noticeable large vortices. This problem is of considerable importance in regard to large stacks, cables, and bridges.

After nearly a week of continual storm conditions, the two-foot pile broke in the middle seat of a joint about three feet below the lower clamp. An examination of the break indicated a fatigue failure.

Coefficient of Drag

Examples are given in Tables I and II of the measured force as the wave

TABLE I - COEFFICIENT OF DRAG

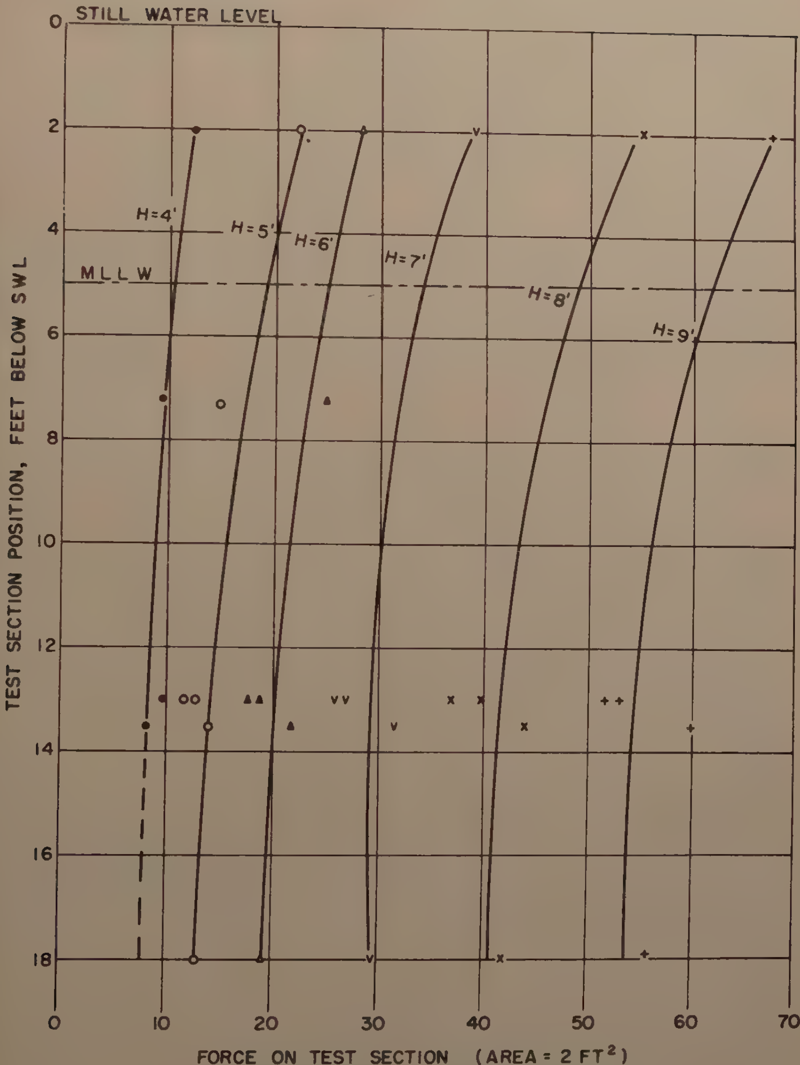
21 December 1953; Roll 24B; 12 3/4" Test Pile; S = 42.5'; d = 49.3' to 46.0'

WAVE	F_C Lbs.	H Ft.	T Sec.	C_D	Re $\times 10^{-5}$	WAVE	F_C Lbs.	H Ft.	T Sec.	C_D	Re $\times 10^{-5}$
228	6.8	7.0	16.5	0.8	2.14	277	6.4	9.2	15.1	0.4	2.93
229	14.4	9.6	16.8	0.9	2.95	278	10	7.9	14.6	0.9	2.44
230	14.0	7.6	17.1	1.5	2.28	279	15	9.6	15	0.9	2.98
231	6.0	6.6	16.1	0.8	2.02	280	4	7.6	14.4	0.4	2.34
232	18.0	8.8	15.5	1.3	2.77	281	10.4	7.8	14.0	0.9	2.42
233	7.0	8.8	15.0	0.5	2.75	282	8.6	7.4	14	0.9	2.31
234	9.2	6.0	14.6	1.4	1.87	283	12	8.6	15.3	0.9	2.68
235	34	9.0	15.3	2.3	2.78	284	24	9.2	12.5	1.6	1.65
236	40	13.6	15.9	1.2	4.17	285	22.6	9.0	15.6	1.6	2.79
237	56	17.6	15.4	1.0	5.44	286	25	9.0	16.3	1.8	2.75
238	58	20.5	15.2	0.8	6.30	287	7.2	7.7	16.5	0.7	2.38
239	62	17.8	15.6	1.1	6.34	288	13.4	9.0	16.3	1.0	2.75
240	21.0	10.1	16	0.5	3.12	289	14.6	8.9	15.6	1.0	2.77
241	10.0	8.0	16	0.9	2.47	290	22	12.5	14.3	0.8	3.90
242	9.2	5.4	15.5	1.3	2.00	291	12.4	7.8	18.6	1.1	2.48
243	12.4	8.5	13.8	0.9	2.67	292	20	7.0	14.8	2.2	2.17
244	21.0	9.2	14.8	1.4	2.85	293	28	9.8	14.5	1.6	3.04
245	12.0	7.1	10.5	1.3	2.25	294	26	9.2	14.5	1.7	2.84
246	15.2	9.3	14.3	1.0	2.86	295	24	9.3	13	1.6	2.92
247	22.8	8.4	14.0	1.8	2.64	296	12	9.0	15.3	0.8	2.78
248	21.2	6.3	14.0	3.0	1.96	297	10	6.6	15	1.3	2.07
249	10.2	6.6	13.0	1.0	2.07	298	16	7.8	14	1.5	2.42
250	9	8.6	12.7	0.7	2.65	299	17	9.0	15.4	1.2	2.80
251	12	8.9	12.8	0.8	2.78	300	52	14.5	15.4	1.4	4.46
252	14	9.6	12.8	0.9	3.01	301	54	17.0	14.8	1.0	5.28
253	6.6	6.6	12.4	0.9	2.08	302	46	13.5	15.3	1.4	4.14
254	15.2	9.2	16.1	1.1	2.82	303	8	8.0	13.8	0.7	2.50
255	22	8.6	14.6	1.6	2.68	304	10	7.6	15.0	1.0	2.38
256	22	8.6	14.8	1.6	2.66	305	26	11.5	15.3	1.1	3.56
257	8.8	5.6	14.8	1.5	1.76	306	58	19.0	15.0	0.9	5.88
258	8.6	8.0	14.8	0.7	2.48	307	48	15.0	14.5	1.2	4.64
259	7.8	8.3	16.1	0.7	2.52	308	44	13.0	13.5	1.4	4.09
260	7.2	8.2	14.0	0.6	2.56	309	11	6.6	13.7	1.4	2.08
261	10	5.3	14.0	2.0	1.65	310	14	8.0	13.0	1.2	2.53
262	13.2	7.0	15.4	1.5	2.18	311	17	9.3	13.0	1.1	2.92
263	6.6	4.4	15.8	1.7	1.37	312	15	8.4	14.0	1.2	2.62
264	8	7.8	15.0	0.7	2.45	313	16	8.6	12.0	1.2	2.75
265	25.2	10.2	15.0	0.9	3.18	314	8	7.0	14.0	0.8	2.30
266	40	12.6	15	1.4	3.89	315	10	8.0	14.3	0.8	2.54
267	54	13.3	15.5	1.7	4.08	316	18	9.7	13.8	1.0	3.07
268	36	13.6	15	1.1	4.22	317	13.6	8.5	13.2	1.0	2.70
269	40	11.5	14.5	1.5	3.58	318	10	7.0	14.7	1.1	2.23
270	30	9.4	13.7	1.9	2.94	319	8	6.5	13.7	1.0	2.08
271	28	11.6	14.5	-	3.00	320	10	8.0	14.1	0.9	1.78
272	10	5.8	15	1.6	1.82	321	17	8.2	14.8	1.3	2.63
273	10	9.0	15	0.7	2.83	322	12.4	8.9	14.2	0.8	2.85
274	16	9.2	15	1.0	2.89	323	9.2	6.0	15.1	1.1	1.92
275	6	7.0	15.4	0.7	2.66	324	7.6	6.0	14.5	1.1	1.92
276	12	8.3	15.4	1.0	2.56	325	12	5.5	14.0	2.0	1.76

TABLE II - COEFFICIENT OF DRAG

13 February 1954; Roll 51; 24" Test Pile; S = 33.0'; d = 48.0' to 46.0'

WAVE	F_C Lbs.	H Ft.	T Sec.	C_D	Re $\times 10^{-5}$	WAVE	F_C Lbs.	H Ft.	T Sec.	C_D	Re $\times 10^{-5}$
23	25.5	9.8	15.0	0.8	5.75	73	23.2	9.5	13.2	0.8	5.51
24	29.0	12.0	15.2	0.6	7.05	74	29.0	10.0	15.4	0.9	5.65
25	11.6	10.1	12.8	0.3	5.87	75	5.8	9.0	12.3	0.2	5.22
26	41.6	8.6	12.1	1.7	5.03	76	17.4	8.2	13.9	0.8	4.81
27	11.6	9.0	13.8	0.4	5.29	77	17.4	6.6	11.9	1.2	3.79
28	11.6	9.3	12.2	0.4	5.45	78	11.6	6.0	12.0	1.0	3.44
29	29.0	10.8	14.3	0.7	6.37	79	17.4	9.2	13.2	0.6	5.27
30	5.8	11.4	13.2	0.1	7.21	80	29.0	10.3	13.5	0.8	5.99
31	11.6	9.0	13.5	0.4	5.20	81	29.0	10.6	11.8	0.8	6.08
32	11.6	7.0	14.6	0.7	4.11	84	17.4	7.5	12.6	0.9	4.33
33	29	7.8	14.3	1.5	4.46	85	40.6	10.5	9.1	1.2	5.88
34	26.1	8.9	13.5	1.0	5.10	86	34.8	10.0	13.1	1.1	5.73
35	23.2	9.0	14.4	0.9	5.19	87	17.4	7.5	13.5	0.9	4.40
36	5.8	12.2	13.6	0.1	7.05	88	26.2	11.5	13.7	0.6	6.75
37	11.6	8.5	11.6	0.5	4.84	90	14.5	7.8	13.8	0.7	4.54
38	11.6	10.0	11.2	0.4	5.65	91	14.5	8.1	12.0	0.7	4.75
39	11.6	7.3	13.0	0.7	4.17	92	11.6	6.5	13.6	0.8	3.83
40	11.6	9.0	12.9	0.4	5.20	93	17.4	8.0	9.6	0.9	4.54
41	11.6	6.1	14.4	1.0	3.51	94	5.8	10.2	14.4	0.2	5.99
42	11.6	11.5	14.0	0.3	6.80	96	17.4	9.5	12.3	0.6	5.55
43	40.6	8.0	13.7	2.0	4.60	97	11.6	9.1	15.2	0.4	5.32
44	20.3	9.5	15.2	0.7	5.45	98	17.4	10.6	14.7	0.5	6.30
45	17.4	8.6	14.7	0.7	4.98	99	34.8	7.3	14.8	1.9	4.28
46	11.6	8.1	15.4	0.5	4.70	101	17.4	9.1	13.6	0.6	5.28
47	11.6	8.0	13.2	0.6	4.75	102	17.4	8.2	14.0	0.8	4.80
48	11.6	8.5	12.0	0.5	4.83	103	5.8	8.0	13.5	0.3	4.70
49	29.0	8.5	13.6	1.2	4.85	104	29.0	10.5	15.2	0.8	6.32
51	23.2	7.5	15.3	1.3	4.33	105	29.0	9.2	11.7	1.0	5.32
52	31.9	10.2	12.2	1.0	5.84	106	34.8	10.0	15.0	1.0	5.88
53	29.0	10.0	13.0	0.9	5.81	107	23.6	7.5	14.2	1.2	4.36
54	11.6	6.0	12.0	1.0	3.42	108	34.8	10.3	14.5	1.0	6.06
55	11.6	11.5	14.2	0.3	6.86	109	-	-	-	-	-
56	40.6	11.0	14.4	1.0	6.29	110	11.6	9.3	17.0	0.4	5.47
57	23.2	12.5	14.3	0.5	7.29	111	5.8	8.5	11.2	0.2	4.98
58	34.8	11.5	13.5	0.8	6.60	112	8.7	7.0	9.8	0.6	3.99
59	31.9	12.2	15.0	0.7	7.06	113	17.4	7.3	12.7	1.0	4.30
60	23.2	8.0	14.4	1.1	4.62	114	34.8	9.0	15.2	1.3	5.32
61	11.6	8.0	12.0	0.6	4.62	116	5.8	8.3	13.1	0.3	4.86
62	20.3	5.2	10.0	2.4	2.94	117	40.6	10.8	14.1	1.0	6.30
63	23.2	9.0	12.2	0.9	5.20	118	20.3	11.5	11.0	0.5	6.58
64	23.2	8.5	14.3	1.0	4.87	119	17.4	7.5	11.4	0.9	4.36
65	17.4	6.2	15.7	1.4	3.55	120	17.4	11.0	12.0	0.4	6.39
66	40.6	9.5	11.0	1.4	5.41	121	26.2	7.0	13.7	1.6	4.07
67	17.4	8.0	11.0	0.9	4.52	122	23.2	9.6	12.8	0.8	5.61
68	17.4	8.5	11.8	0.7	4.99	123	29.0	7.5	14.8	1.5	5.13
69	17.4	6.0	12.5	1.5	3.46	124	34.8	8.1	13.6	1.6	4.76
70	11.6	9.0	13.9	0.4	5.25	125	29.0	10.0	15.3	0.9	5.89
71	17.4	8.5	16.2	0.7	5.00	126	20.3	11.0	14.5	0.7	6.44
72	8.7	9.2	15.0	0.3	5.30	127	31.9	10.0	15.3	1.0	5.89



Data from Rolls 43-45, 51-55 ; water depth = 51 ft. in all cases;
$T_{ave} \cong 13$ sec. in all cases

DISTRIBUTION OF FORCE WITH RESPECT TO DEPTH

HYD-6939

FIGURE 10

crests passed the test piles, together with wave heights, wave periods, computed coefficients of drag, and Reynolds number. In Figure 11 the values of C_D are plotted with respect to Reynolds number. The data from experiments by Texas A & M[22] are also plotted in this figure for comparison.

Because the engineer is interested in the effect of the larger waves, a graph of C_D vs. Re was prepared using the data obtained from waves greater than ten feet in height (Figure 12). It can be seen that the data show no different trend than is the case for all of the data presented in Figure 11.

There is considerable scatter in the data shown in Figures 11 and 12, but this is to be expected considering the variation in degree of turbulence that exists in the vicinity of a pier, the variation of the waves from those described by theory, and the fact that locally generated wind waves often existed in addition to the large storm waves. When considering the amount of scatter, one should keep in mind the difficulty encountered in correlating aerodynamic data from several wind tunnels, even though the control is far superior to that which can possibly be obtained in nature. Dryden and Abbott state:[9]

> "The use of wind-tunnel data for predicting the flight performance of aircraft has always been hampered by the presence of turbulence in the air stream. Comparison of results obtained on spheres in the wind tunnels of Prandtl and Eiffel in 1912 showed that turbulence could have gross effects on aerodynamic measurements comparable with the effects of Reynolds number. Such results led to the establishment of international programs of tests of standard airfoil and airship models and to numerous comparative tests of spheres in wind tunnels of different turbulence. It is now known that the drag of a sphere may vary by a factor as large as 4, the minimum drag of an airship airfoil model by a factor of at least 2, and the maximum lift of an airfoil by a factor of as much as 1.3 in air streams of different wind tunnels at the same Reynolds and Mach numbers."

A word of caution should be made in regard to the trend of the coefficient of drag versus Reynolds number. Fage and Warsap[11] have shown that the effect in surface roughness is to cause the coefficient of drag to remain much higher at Reynolds numbers in the critical and supercritical region ($Re > 2 \times 10^5$). Delany and Sorensen[7] found in wind tunnel tests on circular cylinders that the coefficient of drag increased with increasing Reynolds number for $R_e > 4 \times 10^5$.

Coefficient of Mass

Examples are given in Table III of the measured force as the still-water level (leading the crest) of the waves passed the test pile, together with wave heights, wave periods and computed coefficients of mass.

In Figure 13 are presented values of C_M. These have been plotted on statistical graph paper as a function of the cumulative precentage of values lying below the value in question. It can be seen that the points form a straight line in the major range of interest, indicating that the scatter of data about the mean is a normal Gaussian distribution. The mean value is 2.5, the standard deviation 1.2, and the skewness approximately zero. The mean value is somewhat greater than the theoretical value of 2.0[14] for a perfect fluid.

The coefficient of mass was plotted as a function of the Reynolds number,

TABLE III - COEFFICIENT OF MASS

13 February 1954; Roll 51; 24" Test Pile; S = 33'; d varied from 48' to 46'

WAVE	F_{SWL} Lbs.	H Ft.	T Sec.	C_M	WAVE	F_{SWL} Lbs.	H Ft.	T Sec.	C_M	WAVE	F_{SWL} Lbs.	H Ft.	T Sec.	C_M
46	-	8.1	15.4	-	96	40.6	9.5	12.3	3.3	146	27.8	7.6	14.3	3.3
47	11.6	8.0	13.2	1.2	97	29.0	9.1	15.2	3.0	147	13.9	10.0	12.4	1.1
48	23.2	8.5	12.0	2.1	98	29.0	10.6	14.8	2.5	148	48.1	9.3	10.1	3.2
49	23.2	8.5	13.5	2.3	99	17.4	7.3	14.8	2.2	149	29.0	8.2	14.8	3.2
50	29.0	11.5	14.5	2.3	100	52.3	11.0	14.0	4.1	150	34.8	9.0	12.5	3.2
51	23.2	7.5	15.3	3.0	101	17.4	9.1	13.6	1.6	151	58.0	11.0	12.9	4.3
52	17.4	10.2	12.2	1.3	102	34.8	8.2	14.0	3.6	152	46.4	10.0	14.1	4.0
53	52.2	10.0	13.0	4.3	103	23.2	8.0	13.5	2.4	153	48.7	9.5	14.6	4.6
54	11.6	6.0	12.0	1.5	104	23.2	10.5	15.2	2.1	154	81.2	10.6	14.0	6.6
55	46.4	11.5	14.2	3.5	105	23.2	9.2	11.7	1.8	155	30.7	8.0	12.2	3.0
56	34.8	11.0	14.4	2.9	106	31.9	10.0	15.0	2.9	156	40.6	8.6	12.2	3.9
57	52.2	12.5	14.3	3.7	107	29.0	7.5	14.2	3.4	157	44.1	11.3	13.4	3.3
58	43.5	11.5	13.5	3.2	108	40.6	10.3	14.5	3.5	158	13.3	8.7	15.2	1.4
59	40.6	12.2	15.0	3.0	109	17.4	12.0	16.9	1.5	159	31.9	10.2	13.9	2.7
60	11.6	8.0	14.4	1.6	110	23.2	9.3	17.0	2.5	160	26.1	7.0	13.0	3.0
61	26.1	8.0	12.0	2.5	111	11.6	8.5	13.7	1.4	161	31.3	10.4	13.0	2.4
62	11.6	5.2	10.0	1.4	112	23.2	7.0	9.8	2.1	162	39.5	7.0	12.8	4.5
63	26.1	9.0	12.2	2.2	113	29.0	7.3	12.7	3.2	163	27.3	10.2	13.4	2.2
64	29.0	8.5	14.3	3.0	114	34.8	9.0	15.2	3.6	164	26.7	6.0	14.0	3.8
65	29.6	6.2	15.7	4.7	115	13.4	9.0	15.1	1.4	165	34.8	7.6	14.7	4.1
66	37.7	9.5	11.0	2.8	116	31.9	8.3	13.1	3.1	166	26.7	10.0	14.8	2.4
67	17.4	8.0	11.0	1.5	117	12.8	10.8	14.1	1.0	167	23.2	10.8	15.4	2.0
68	31.9	8.5	11.8	2.8	118	45.3	11.5	11.0	2.7	168	34.5	9.0	15.2	3.6
69	13.3	6.0	12.5	1.8	119	11.6	7.5	11.4	1.1	169	24.9	9.0	15.8	2.7
70	34.8	9.0	13.9	3.3	120	41.7	11.0	12.0	2.8	170	26.7	7.0	13.8	3.3
71	29.0	8.5	16.2	3.4	121	26.1	7.0	13.7	3.2	171	34.7	8.5	14.5	3.7
72	17.4	9.2	15.0	1.8	122	33.7	9.6	12.8	2.9	172	36.6	10.5	14.6	3.1
73	29.0	9.5	13.2	2.5	123	36.0	7.5	14.8	4.3	173	34.7	10.6	14.9	3.0
74	40.6	10.0	15.4	3.9	124	45.3	8.1	13.6	4.8	174	53.9	9.2	15.5	5.6
75	11.6	9.0	12.3	1.0	125	29.0	10.0	15.3	2.7	175	17.4	6.0	12.3	2.2
76	31.9	8.2	13.9	3.3	126	30.2	11.0	14.5	2.5	176	23.2	5.6	14.3	3.7
77	29.0	6.6	11.9	3.3	127	36.0	10.0	15.3	3.4	177	30.2	8.0	13.0	3.1
78	37.7	8.0	12.0	3.8	128	37.7	8.8	15.7	4.1	178	31.9	7.1	11.3	3.2
79	27.8	9.2	13.2	2.5	129	18.6	10.0	12.8	1.5	179	40.6	8.0	13.3	4.2
80	37.7	10.3	13.5	3.1	130	29.0	10.0	13.5	2.5	180	38.9	10.0	12.4	3.0
81	29.0	10.6	11.8	2.0	131	17.4	8.0	13.3	1.8	181	49.3	9.5	12.2	3.6
82	11.6	8.0	11.2	1.0	132	30.1	11.5	13.0	2.1	182	17.4	9.6	13.7	1.5
83	23.2	6.2	10.4	2.4	133	40.6	10.2	13.7	3.7	183	39.4	10.0	10.1	2.4
84	29.0	7.5	12.6	3.1	134	36.5	9.3	11.7	2.8	184	27.9	5.1	8.7	2.9
85	34.8	10.5	9.1	2.0	135	31.9	6.8	11.4	3.4	185	20.3	8.0	13.7	2.2
86	46.4	10.0	13.1	3.8	136	29.0	8.3	12.9	2.8	186	31.9	9.5	14.6	3.0
87	23.2	7.5	13.5	2.5	137	26.1	7.0	13.2	3.0	187	39.5	10.1	12.4	3.0
88	40.6	11.5	13.7	3.0	138	16.3	9.0	13.5	1.5	188	23.2	11.0	12.0	1.6
89	75.5	11.3	11.2	4.3	139	55.2	11.6	13.8	4.0	189	20.3	11.2	14.5	1.7
90	29.0	7.8	13.8	3.2	140	29.0	11.2	15.4	2.4	190	23.2	10.0	15.1	2.2
91	34.8	8.1	12.0	3.2	141	40.6	11.5	15.1	3.3	191	40.6	9.0	15.1	4.2
92	26.1	6.5	13.6	3.4	142	22.1	9.5	14.5	2.1	192	8.7	8.8	14.0	0.9
93	29.0	8.0	9.6	2.2	143	17.4	7.8	12.4	1.7	193	29.0	9.0	15.4	3.0
94	37.7	10.2	14.4	3.3	144	46.4	9.5	12.8	3.9	194	37.7	9.3	12.5	3.2
95	29.0	9.6	13.4	2.6	145	15.7	9.0	14.1	1.5	195	24.9	10.0	11.3	1.8

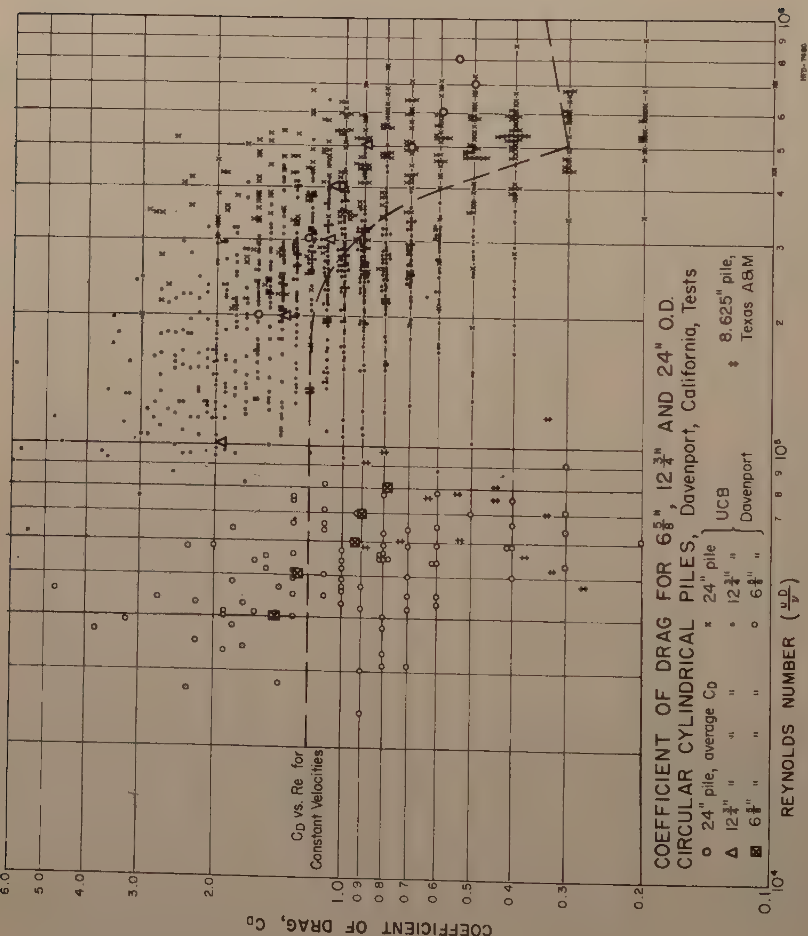
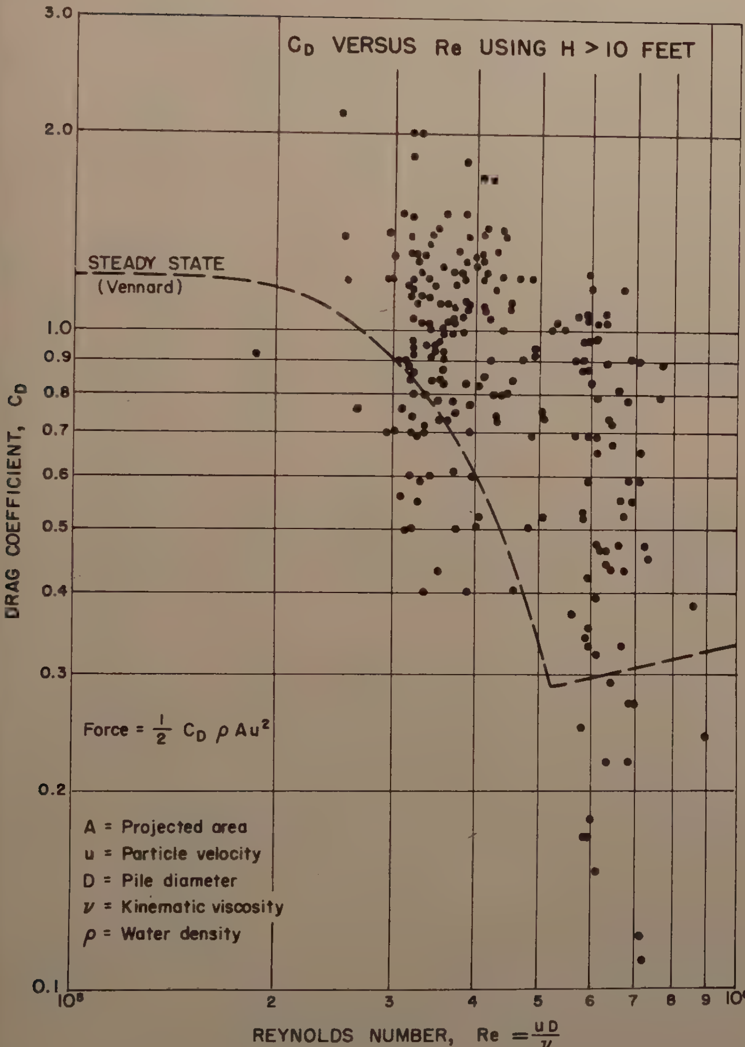

COEFFICIENT OF DRAG FOR 6⅝", 12¾" AND 24" O.D. CIRCULAR CYLINDRICAL PILES, Davenport, California, Tests
o 24" pile, average C_D
Δ 12¾" " " "
⊠ 6⅝" " " "
x 24" pile
• 12¾" "
o 6⅝" "
UCB
Davenport
‡ 8.625" pile, Texas A&M
C_D vs. Re for Constant Velocities
COEFFICIENT OF DRAG, C_D
REYNOLDS NUMBER $\left(\frac{uD}{\nu}\right)$

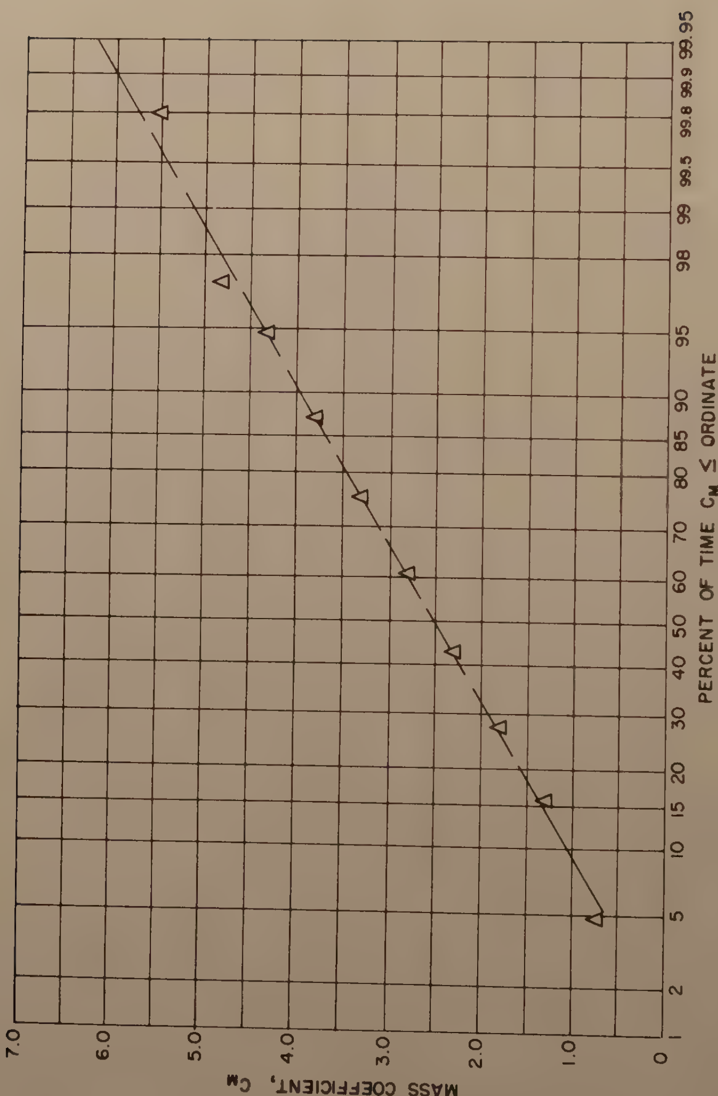
FIGURE 11



COEFFICIENT OF DRAG AS A FUNCTION OF REYNOLDS NUMBER FOR WAVES HIGHER THAN 10 FEET

HYD-6941

FIGURE 12



Values from Rolls 41, 43, 51 & 53, 400 waves

DISTRIBUTION OF VALUES OF C_M

HYD-6942

FIGURE 13

where Reynolds number was computed using the maximum horizontal component of velocity for the particular wave for which the value of C_M was obtained. It was thought that the coefficient of mass should exhibit some relationship with the flow conditions, as represented by Reynolds number; however, this was not the case, at least within the limits of experimental error and the range of experimental conditions. In addition, values of the coefficient of mass were plotted as a function of the values of the coefficient of drag, for the same waves and again no apparent relationship was noticed.

After the original analyses had been made it was suggested that a relationship might exist between C_M and $\partial u/\partial t$. As can be seen in Figure 14B, this is apparently not the case, at least within the range of experimental conditions. The same data that showed a relationship between C_M and $\partial u/\partial t$ also showed a definite relationship between C_M and wave period. As can be seen in Figure 14A, this was not the case for the Davenport data. The two sets of C_M were determined in a different manner, however, and it is not surprising that the same relationships were not found.

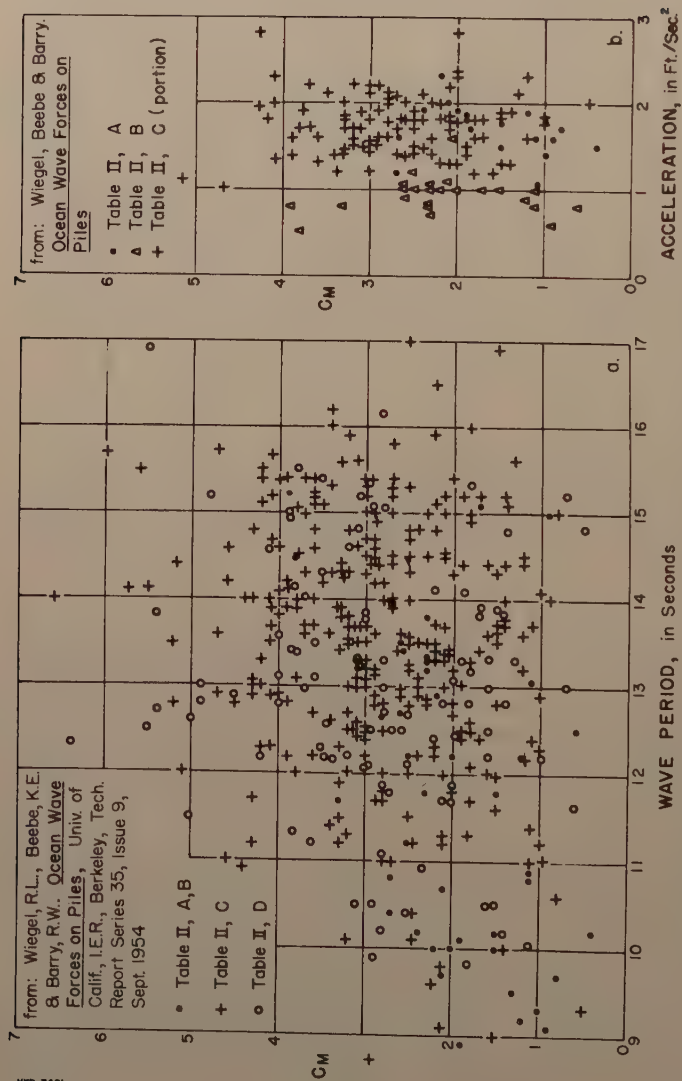
Resistance Coefficient

A few wave and force records were analyzed after the manner of Crooke(6) and the results plotted in Figure 15. The average curve of Crooke's data is shown also. Crooke's curve can be seen to form an upper limit to the few Davenport data. In addition, the curves obtained by Keim(16) for vertically accelerated cylinders are given for comparison purposes.

General

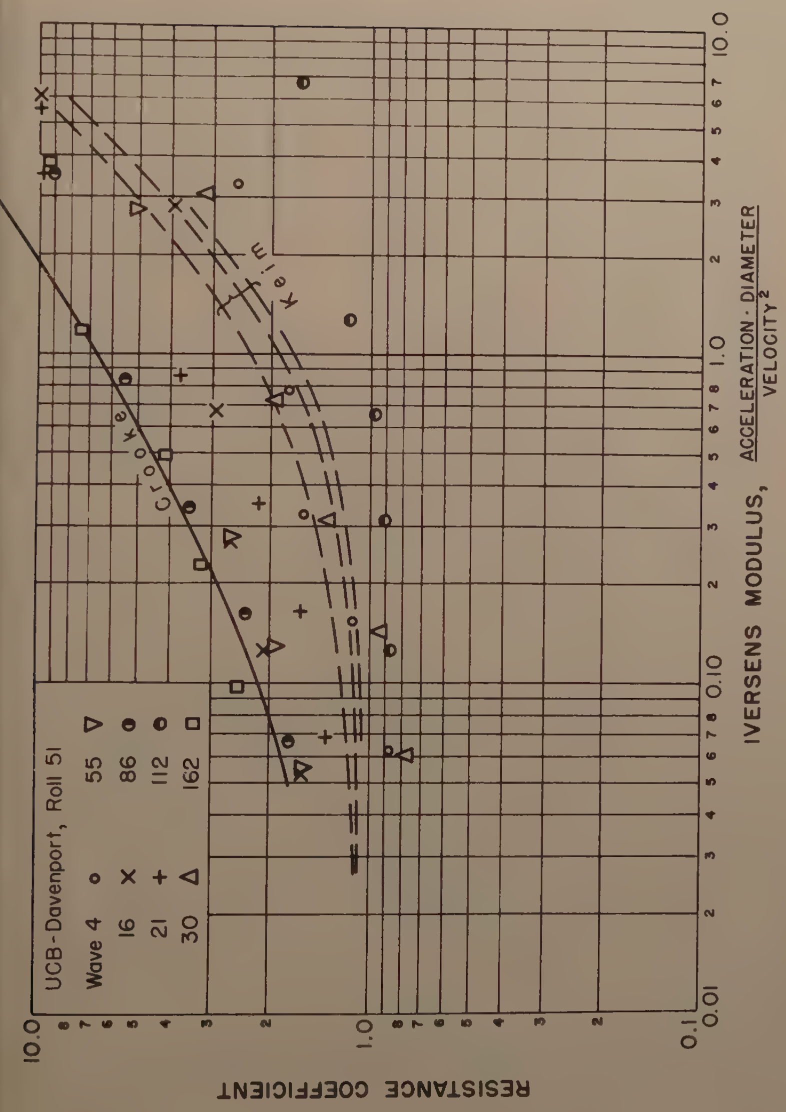
As was mentioned briefly in a previous section, the Morison, O'Brien, Johnson and Schaaf theory(17) for forces exerted by water gravity waves on piles involves several rather severe assumptions. They are that (1) the linear theory for waves correctly predicts wave behavior; (2) the equation for fluid drag and inertia forces on a pile in rectilinear flow hold for oscillatory flow; and (3) the equations for drag and inertia forces may be added linearly to obtain the total force.

The work of Morison and Crooke(19) shows that the linear theory predicts the characteristics of wave shape, particle paths, particle velocities and particle accelerations for uniform waves in the laboratory fairly accurately in water deeper than $d/L = 0.2$. An examination of the data in Reference 19 indicates that the linear theory equation for particle velocity is no less reliable than the more complex equations of Stokes. The more complex equations do show that the horizontal component of particle velocity is greater under the crest than under the trough; however, the simple equation predicts the horizontal component of particle velocity under the crest more reliably on the whole than do the more complex equations; in addition, if the surface time history of the wave is known this theory can be used in the manner of the present paper (Equation 4) to predict a greater velocity at the crest than at the trough. This is important as it is the force exerted by the wave in the direction of wave travel that is of more interest to the engineer, because it is larger than the force exerted on the pile when the pile is in the wave trough. (Note that the data in Figure 7 show a higher force leading the crest than leading the trough of the waves.) In addition, the more complex equations of Stokes are not valid for shallow water since the series which they approximate "blows up."(13)



COEFFICIENT OF MASS *VERSUS* WAVE PERIOD AND HORIZONTAL COMPONENT OF WATER PARTICLE ACCELERATION

FIGURE 14



COMPARISON OF UCB-Davenport, CROOKE AND KEIM RESISTANCE COEFFICIENTS FOR RIGHT CIRCULAR CYLINDERS

HYD-7482

FIGURE 15

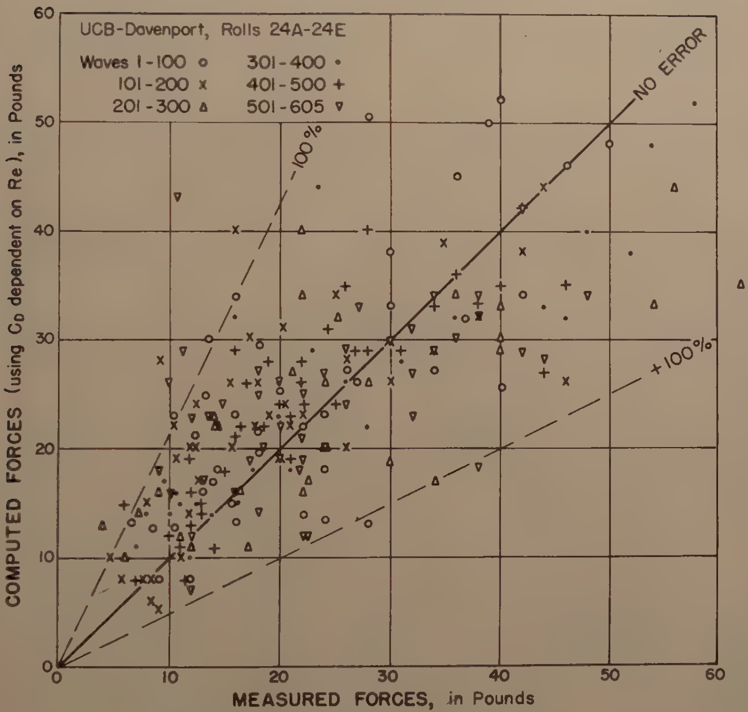
The main disadvantage of the linear wave theory is in regard to the prediction of the water particle acceleration. In addition to the magnitude changing, the phase angle at which the maximum acceleration occurs shifts towards the crest. The few data given in Reference 10 indicate that the actual shift is greater than theory predicts. This would not introduce much error in the determination of the coefficient of mass as the actual still-water level was used as the reference in obtaining the data. It would, however, introduce an error in predicting the maximum total force that would occur when the inertia force is a significant part of the maximum total force, and it apparently would result in an under-prediction of the maximum total force.

In order to check partially the validity of some of the assumptions made in the wave force theory and the analysis of the data, the average values of C_D versus Reynolds number and the average value of C_M (2.5) were used to predict the maximum wave force on the 12-3/4 inch and the two-foot diameter piles. These values were then compared with the maximum measured forces (Figures 16 and 17). The scatter of the predictions with respect to the "no error" line is of the order of plus or minus 100% for both piles. However, the predicted forces are higher than the measured forces for low measured forces and lower than the measured forces for the high measured forces: this is especially true in the case of the two-foot diameter pile (Figure 17). An examination of the records from the standpoint of vibrations indicate that this difficulty is not due to vibrations, per se, although it might be due to a somewhat similar problem. This type of variation cannot be explained by currents, mass transport or a shift in the "zero force line" as these would cause a translation of the points in Figures 16 and 17 in one direction or the other, rather than a rotation.

A typical example of a measured force that was approximately twice as great as the computed force is shown in Figure 7B, Wave No. 146. It can be seen that the force rises rapidly and then drops very rapidly. This type of force often caused the pile to vibrate severely, although in the example shown the vibrations are quite small. A force trace of this type could be caused by the superposition of wind waves (which were present—See Figure 7B) on the swell with a critical phase relationship; it could be caused by the shifting of the phase of the water particle accelerations with respect to wave period; it could be a transient phenomenon due to the higher harmonics which are apparent in the wave records; it could be caused by an eddy hitting the pile in such a manner that the eddy and stream velocities were additive; or it could be caused by a combination of any or all of the above-mentioned phenomena.

It appears that the modified linear theory for predicting wave forces, as presented herein, is only an approximation, and that serious questions arise in regard to the results. A comparison of predicted and measured to maximum wave forces, (including some too high to fit on Figures 16 and 17), indicate that within the range tested a safety factor of 2 to 2.5 should be used, if average values of C_D and C_M are used in the computations. It is believed desirable to use higher values of C_D and C_M, however than the averages, and to adjust the safety factor accordingly.

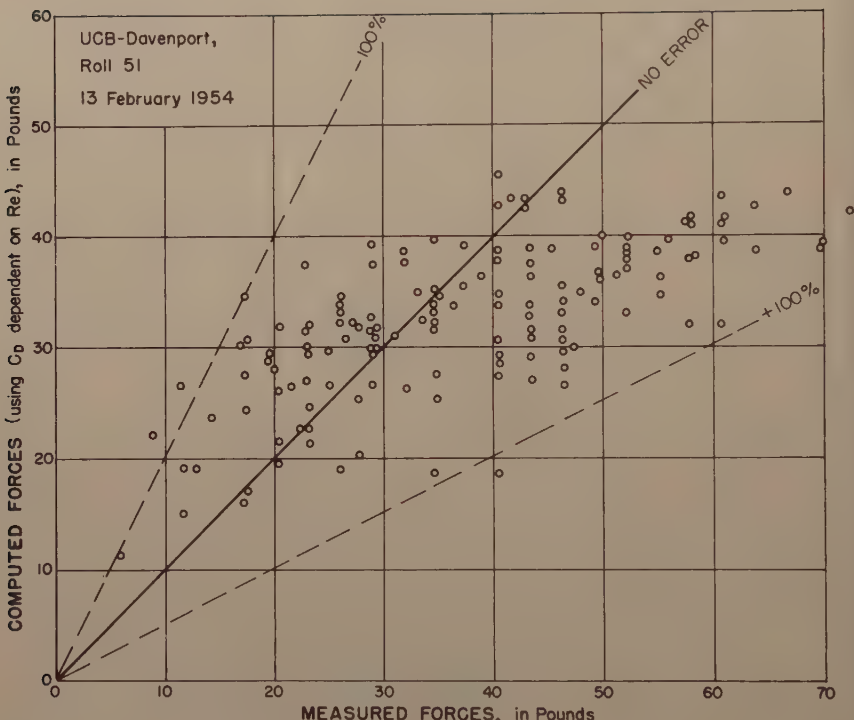
Although considerable scatter was observed in the comparison of predicted and measured maximum forces, and a trend for under-prediction was observed for the higher measured forces, the results are rather remarkable when the differences between the actual conditions (wind waves on top of swell, extreme turbulence—Figure 6, waves in shallow water, eddies, etc.) and the conditions expressed in the equations are considered.



MAXIMUM HORIZONTAL FORCES ON A 12¾" PILE

HYD-7483

FIGURE 16



MAXIMUM HORIZONTAL FORCES ON A
2-FOOT DIAMETER PILE

HYD-7482

FIGURE 17

CONCLUSIONS

1. The force meter and test pile set-up performed as expected and were satisfactory. Vibration pickup on the force records were considerably less than they were in the case of the hinged pile. When the motion of the pile through the water became so great as to create a secondary force, the bottom of the test pile was guyed, and the problem of motion was largely overcome.
2. The distribution of force with distance below the still-water surface was approximately as predicted by the simple theory. As was expected from a study of the work of Morison and Crooke(19) the difference between predicted and measured values was greatest in the range between the trough and the crest of the wave.
3. Tests, performed in such a manner that the test section was exposed in the trough and then submerged by the crest, gave no evidence of impact force.
4. The coefficients of drag, determined for the range of Reynolds numbers between 3×10^4 and 9×10^5, were of the same order of magnitude as for the curve for steady state rectilinear flow. The scatter was not extreme when one considers the great variation in the turbulence of the flow, the pile roughness, eddies from other piles, and the fact that the pile is subject to its own eddies at the leading edge as the flow reverses. A comparison of the C_D for the several pile sizes indicates that there is a secondary effect of pile diameter.
5. The coefficient of mass compared favorably with the theoretical value for a perfect fluid (2.0), and the median value was found to be 2.5. No relationship was found between the coefficient of mass and the coefficient of drag, Reynolds number, horizontal component of local water particle acceleration, or the wave period.
6. Comparison of predicted maximum forces and measured maximum forces, using the average values of C_D versus Reynolds number and the average value of C_M in the modified linear theory for predicting wave forces showed serious discrepancies. However, for the range of conditions tested a safety factor of 2 to 2.5 should be sufficient.
7. After many days of continual action by large waves on the two-foot test section, the six-inch central pile broke in the middle seat of the thread about three feet below the bottom clamp. An examination of the break indicated a fatigue failure.

ACKNOWLEDGMENTS

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LIST OF SYMBOLS

A = projected area perpendicular to particle motion (feet^2)

C = coefficient of resistance (dimensionless)

C_D = coefficient of drag (dimensionless)

C_M = coefficient of mass (dimensionless)

d = still-water depth (feet)

D = pile diameter (feet)

F = total force exerted by wave on pile (pounds)

F_D = drag force on pile (pounds)

F_M = inertia force on pile (pounds)

g = acceleration of gravity (feet per second^2)

H = wave height (feet)

L = wave length (feet)

M_o = mass of the fluid displaced by the pile (pound-seconds^2 per foot)

M_v = added mass (pound-second^2 per foot)

Re = Reynolds number, Du/ν (dimensionless)

S = elevation of water particle above the ocean bottom (feet)

S_c = Elevation above the ocean bottom of the center of the pile test section (feet)

t = time (seconds)

T = wave period (seconds)

u = horizontal component of water particle velocity (feet per second)

V = volume of submerged pile (feet^3)

ΔS = vertical dimension of test section of pile (feet)

dF = increment of force on pile (pounds)

dS = increment of pile length (feet)

$\frac{\partial u}{\partial t}$ = horizontal component of water particle local acceleration (feet per second^2)

ν = kinematic viscosity of sea water (feet^2 per second)

π = 3.1416

ρ = mass density of sea water (pound-second^2 per foot^4)

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HYDRAULICS DIVISION
Proceedings of the American Society of Civil Engineers

THE ESTIMATION OF THE FREQUENCY OF RARE FLOODS

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and Charles J. Smith,[2] A.M. ASCE
(Proc. Paper 1200)

SYNOPSIS

A method of estimating the frequency of annual peak flood flows which uses recorded monthly peak flood flows is suggested. It is shown that the use of monthly peak flood flows approximates the results obtained by using all peak flows but greatly reduces the amount of work involved.

The estimation of the frequency of rare floods is a problem which has long been faced by civil engineers. In the design of spillways, levees, bridges, reservoirs, and many other engineering structures the estimation of the probable frequency of recurrence of floods of different magnitudes is essential to the design of the structure. It has long been recognized that the most fundamental basis for the estimation of flood frequencies on a given watershed is the past record of peak flows on that watershed. Such a record includes all of the fixed and chance factors which are effective in producing floods on a given watershed, such as drainage area, climate, weather, topography, storage, condition of the watershed, and all the lesser factors involved.

Most studies of this type have been based upon the record of peak annual flows, but it has been recognized that there are inherent weaknesses in using these flows which may result in errors of large magnitude in the determination of the recurrence interval of the rarest floods, particularly those which have a recurrence interval greater than the period of record on the stream under study. In some quarters the use of statistical methods for such studies has fallen into complete disrepute. As a result, other methods have been

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suggested which are based upon the discrete study of the individual factors involved and a recombination of these results into an over-all estimate of the probable flood flows. These methods are subject to serious criticism themselves, however, on the grounds that it is impossible in this way to take all the factors into account and some of the factors themselves are not subject to rigorous analysis. Because of this it seems appropriate to re-examine the criticisms of the statistical methods of analysis of annual peak flows to determine which of them are valid and to attempt to eliminate those which are not.

Flood flow studies using the statistical approach are based upon the assumption that the combinations of the numerous factors which produce flood flows are a matter of pure chance and are therefore subject to analysis according to the mathematical theory of probabilities. This leads to the assumption that peak flows of all magnitudes of a given stream will be distributed in frequency according to some normal law of distribution of chance occurrences. If a large number of streams are studied and it is found that their records show a peak flow distribution which follows a particular law of the frequency of occurrence of floods of different magnitudes, the latter assumption is verified.

Numerous investigators long ago noted that logarithms of peak annual flows for a given stream appeared to be distributed according to the laws of chance. Thus when these logarithms were plotted on normal probability paper, the points so plotted appeared to approach a straight line, or, as is more convenient, the plotting of the values themselves on logarithmic probability paper gave the same straight line. It was thus indicated that the logarithms of peak annual flows on a given stream followed the normal laws of chance. On this assumption it was therefore perfectly correct mathematically to compute the frequency of flood magnitudes greater than any which had occurred by applying the mathematical laws of probability or by obtaining the same result graphically by extrapolating the straight line of best fit on the plot of the logarithms of flood flows on probability paper.

Even the earliest proponents of this method soon discovered, however, that the simple assumption of logarithmically normal distribution of peak annual flows appeared to give recurrence intervals too great for rare floods on most watersheds. This was made evident by the fact that in many cases the plotted points definitely plotted in a curve and by the impression that in other cases rare floods of high magnitude on different watersheds were occurring more often than the statistical theory indicated that they should. This indicated that either the basic assumption that the logarithms of peak flows were normally distributed was wrong or that there was some defect in its application. Ignoring this question, many investigators utilized mathematical techniques involving the coefficient of variation and the coefficient of skew to fit skew curves to peak annual flow data that plotted as a curve. This technique becomes little more than a mathematical method of fitting an empirical curve to the data and is little, if any, better than drawing such a curve graphically by eye through the plotted points, and may actually be worse.

Some investigators have, however, pointed out a fundamental weakness of the assumption that peak annual flows are the result of chance factors and are therefore to be expected to follow some type of normal distribution. The defect is that peak annual flows are not a true sample of the universe of occurrences of peak flows during the period of record. If the peak annual flow in any one year is used, it is obvious that a multitude of lesser peaks

are completely ignored in the analysis. It is faulty statistical technique to use selected data in studying any phenomenon which is presumed to follow the basic laws of probability. Such a study should utilize all the data available which is a measure of the phenomenon being considered. Foster,[3] among others, has recognized this and develops an analysis of flood flows upon all peak flows above an arbitrary value. He does not show, however, that if it can be demonstrated that all peak flows follow the logarithmically normal laws of probability, annual peak flows cannot follow such a law but must plot as a curve on logarithmic probability paper. This is due fundamentally to the fact that the smaller the peak flow the greater is its chance of being obscured by a larger flow in any fixed period of time, be it a week, a month, or a year. This is, of course, more apparent and more significant the longer the period of time.

The relation between all the peak flows of record for a given watershed and the annual peak flows for the same watershed can be demonstrated by applying statistical techniques to the recorded peak flows and then applying the laws of probability to determine the chance of any particular value of peak flow being an annual peak flow in the period of record. The use of all peak flows has an added advantage over the use of annual peak flows because the statistical analysis is based upon far more data (about fifty times as much).

For such a study it is necessary to define the term "peak flow." In this study it has been simply defined as any daily twenty-four hour average flow which is larger than that for both the preceding and following day. This eliminates Foster's arbitrary lower limit but introduces certain irregularities at very low peak flows because of the presence of storage effects on the watershed. These are not significant with regard to higher flows, however. In Figures 1 and 2 are shown portions of the plots of the frequency of occurrence of all such peak flows for the Lehigh River at Tannery, Pennsylvania, and the Tunkhannock Creek at Dixon, Pennsylvania. The points for Tannery plot as an excellent straight line, and those for Dixon plot fairly close to a straight line, indicating the possibility that peak flows do follow a logarithmically normal type of distribution. This question is not, however, of basic significance to the problem at hand.

It was early realized that for practical application the use of all peak flows for making a study of the frequency of rare floods would not be desirable because of the very great amount of tedious work involved in assembling the data on all peak flows. It was also realized that it was possible that the use of monthly peak flows, which are a matter of easily accessible record and therefore obtainable with much less effort, might prove to be a satisfactory basic source of information instead of all peak flows. It might be the case that the effect of larger peak flows obscuring smaller peak flows in a time period as short as a month would not be serious at the larger rates of flow involved in large floods. Therefore, the first step in the analysis was to determine the relation between the frequencies of all peak flows and monthly peak flows.

The record available at Tannery covered 406 months and 1,854 peak flows, while that at Dixon covered 408 months and 1,961 peak flows. These records were tabulated to determine the number of months which contained various numbers of peak flows. A theoretical analysis was then made to determine

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3. Edgar E. Foster, Rainfall and Runoff, New York: The Macmillian Company.

the distribution of peak flows by numbers per month for Tannery. A similar distribution for Dixon would have been nearly the same because of the similarity in the total number of months and peaks involved. In Figure 4 is shown the actual distribution for Tannery. One outstanding difference between the theoretical curve and the actual curves is apparent; that is, the theoretical curve is much flatter. It is concluded that the time intervals between peaks are more consistently near the average period of 4.57 and 4.81 peaks per month than would be the case were they dependent on chance alone. An obvious but unverified explanation of this is the known general tendency for the great cyclonic disturbances, which are one of the principal instruments of rainfall and runoff, to cross the country at approximately weekly intervals.

It is obvious that any peak falling in a month with only one peak will be recorded as the peak flow for that month while in months with more than one peak only one will be recorded as a peak, and all the rest will be lost to the record. It is equally true that the higher the peak the less is its chance of being obscured by a still higher one in any one month. The chance of a peak of a certain magnitude and frequency of occurrence being the highest value in any month may be determined by the laws of probabilities. Using the actual distribution of the number of peaks per month, this has been done for Tannery and Dixon. The results are plotted in Figures 1 and 2. These theoretical frequencies have been compared with the actual frequencies of monthly peaks of record by plotting them on the same figures. It may be seen that there is substantial agreement at the higher rates of flow between the theoretical frequencies based upon all peaks and the actually recorded monthly peak flows. Thus it is concluded that for practical purposes for further study an analysis of actual monthly peak flows will give results nearly as good as those obtained obtained by analyzing all peak flows. This is particularly important because of the large amount of time consuming computations necessary in computing these values from peak flow records and the extra time and effort involved in obtaining the records.

While the original frequency study of all peak flows was not related to a specific time period, the monthly peak flow frequencies obtained either from all peaks or from monthly records is now in the framework of a specific time interval, the month. By probability methods it is readily possible to determine the chance that a monthly peak flow of any frequency would have of being a peak annual flow. If P_m is the probability of a given event occurring in any one month, the probability, P_y, that it will occur in a year is much greater and can be expressed as

$$P_y = 1 - (1 - P_m)^{12}$$

If the event under consideration is the occurrence of a peak flow of a certain magnitude or greater, this equation may be applied to the transformation of the probabilities of the occurrence of peak monthly flows to the probability of peak annual flows. The solution of this equation gives the values of P_y shown in Table I for various values of P_m. From the actual values of P_m as plotted or from the line of best fit, a line may be plotted for the probabilities of peak annual flows of various magnitudes. This has been done in Figures 1 and 2. The validity of this technique may be checked by plotting the actual recorded peak flows. This has been done for Tannery and Dixon as shown on these two figures. It may be seen that the line plotted from monthly peak

TABLE I

CHANCES OF ANNUAL OCCURRENCES BASED UPON THE PROBABILITY OF MONTHLY OCCURRENCES

P_m %	P_y %	P_m %	P_y %
.01	0.12	3	30.6
.02	0.24	5	46.0
.05	0.60	7	58.1
.1	1.19	10	71.8
.2	2.4	20	93.1
.5	5.8	30	98.62
	11.4	40	99.78
	21.5	50	99.976

flows is a curve which agrees very well with the actual annual peak flows.

As a further check of this method of establishing a curve which represents the true distribution of peak flows, the monthly peak flows for the Iowa River at Iowa City, Iowa, were analyzed and plotted as shown in Figure 3. The transformation coefficients were then applied to this plot to give the annual peak curve shown. Again this shows excellent agreement with the actual annual peak flows.

In order to demonstrate the great errors in the determination of the recurrence interval of rare floods which are made in assuming that annual peak flows follow the logarithmically normal laws of probability, the straight line of best fit has been computed for the recorded data and plotted in Figures 1, 2, and 3. Referring to Figure 1, it may be seen that the straight line of best fit gives a frequency of occurrence of 0.85% for a 20,000 cfs flood, while the computed curve gives 2.7%. These give recurrence intervals of 118 years and 37 years, respectively. The 1955 hurricane resulted in a momentary peak rate of discharge of 58,300 cfs at Tannery, which is perhaps equivalent to a 24-hour average of 40,000 cfs. From the straight-line plot this would have a recurrence interval of 10,000 years, while the curve would give a recurrence interval of 555 years. It is obvious that differences of this magnitude are of major significance in the design of hydraulic structures.

The technique of using actual monthly peak flows as the basis for determining the distribution of annual peak flows has the following advantages:

1. It bases the results on a much larger amount of basic data.
2. It utilizes data readily available in the Water Supply Papers which require much less clerical work in their organization than does the use of all peak flows.

3. The logarithmic plot of monthly peak flow frequency is so nearly a straight line that reasonable extrapolation can be made with confidence.

4. The resulting plot of the frequency distribution of the logarithms of annual peak flows follows a curve of the type which must exist if the logarithms of all peak flows are distributed according to the laws of chance.

The technique here described is suggested in the belief that it will be useful to engineers who must make economic studies based upon the probability of occurrence of floods larger than any which have occurred in the periods of record. The applicability of the method should be verified by the study of numerous known records. It is suggested that other investigators who have available the necessary data in readily usable form for other watersheds may investigate the technique to determine its general reliability.

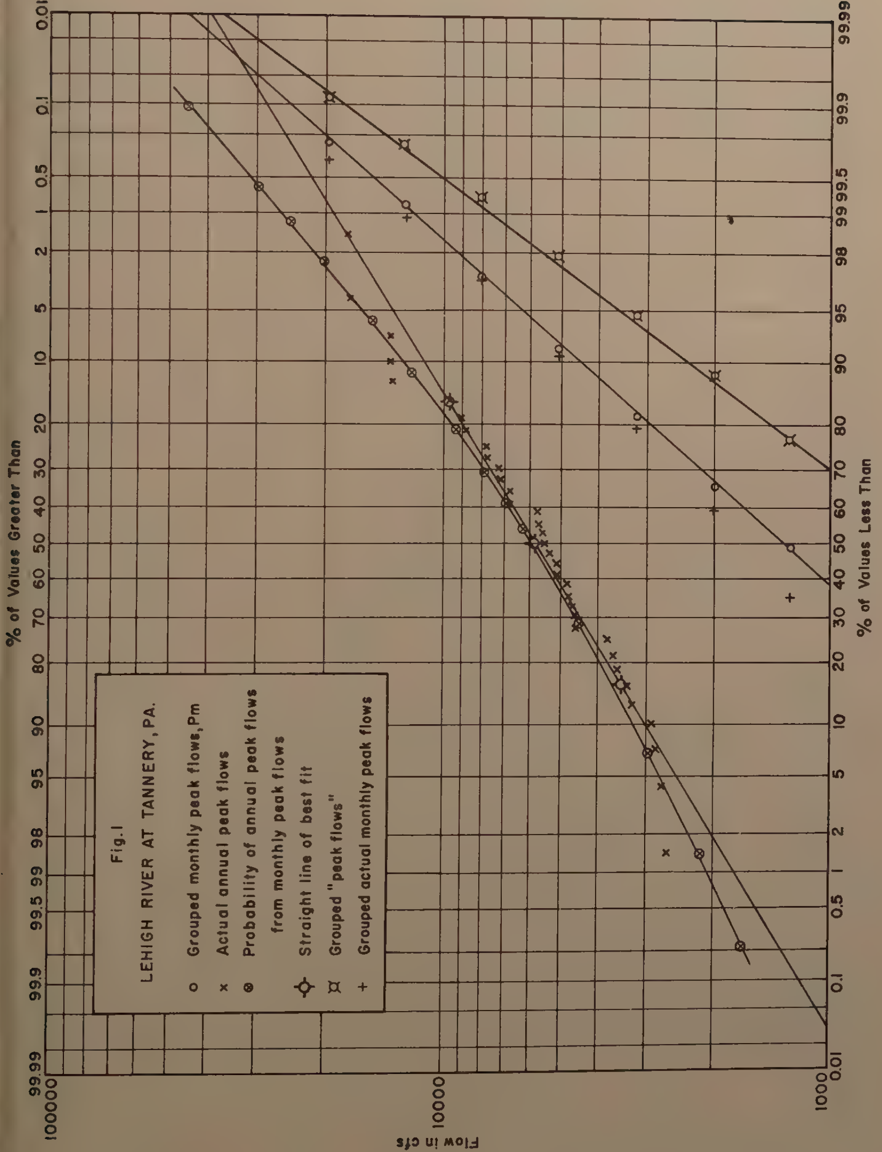


Fig.1
LEHIGH RIVER AT TANNERY, PA.
o Grouped monthly peak flows, Pm
x Actual annual peak flows
⊗ Probability of annual peak flows from monthly peak flows
Straight line of best fit
Grouped "peak flows"
+ Grouped actual monthly peak flows
% of Values Greater Than
% of Values Less Than
Flow in cfs
100000
10000
1000
0.01
0.1
0.5
1
2
5
10
20
30
40
50
60
70
80
90
95
98
99
99.5
99.9
99.99

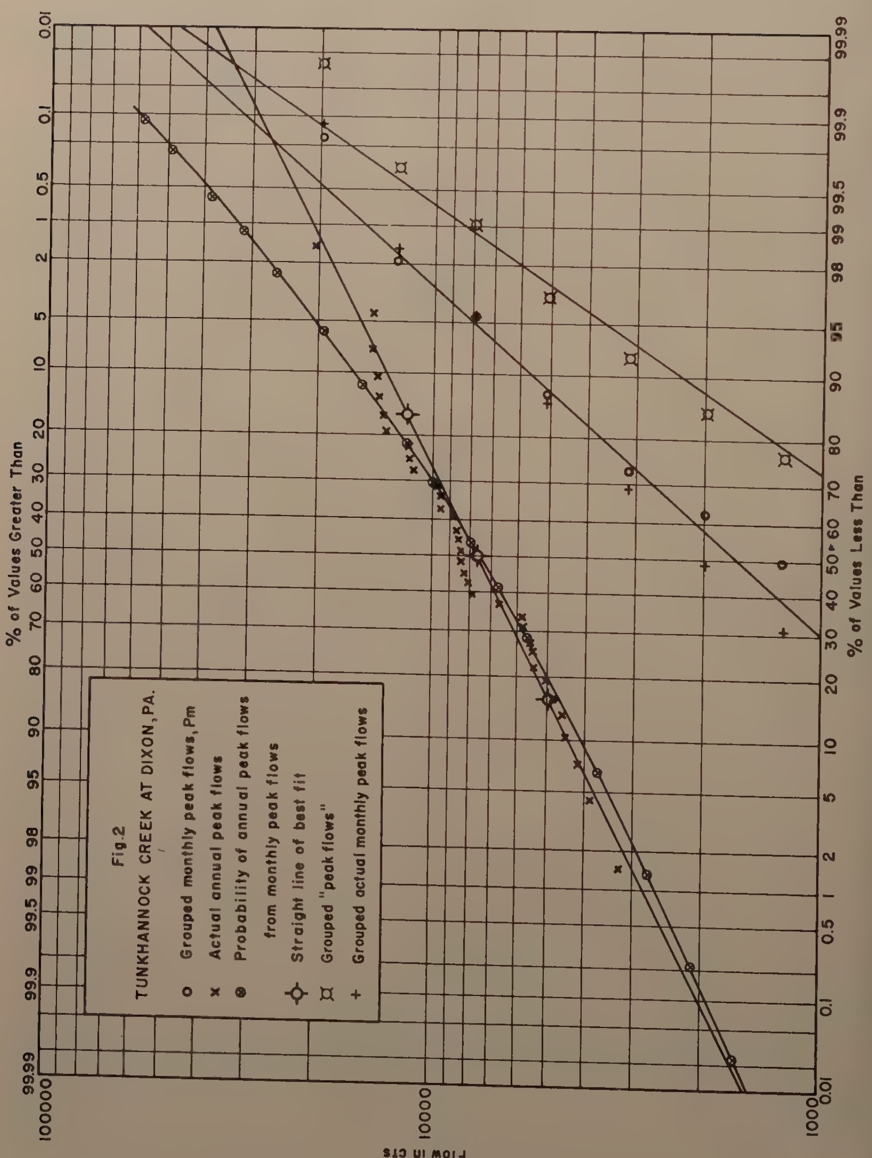


Fig. 2
TUNKHANNOCK CREEK AT DIXON, PA.
Grouped monthly peak flows, Pm
Actual annual peak flows
Probability of annual peak flows from monthly peak flows
Straight line of best fit
Grouped "peak flows"
Grouped actual monthly peak flows
% of Values Greater Than
% of Values Less Than
Flow in cfs
100000
10000
1000

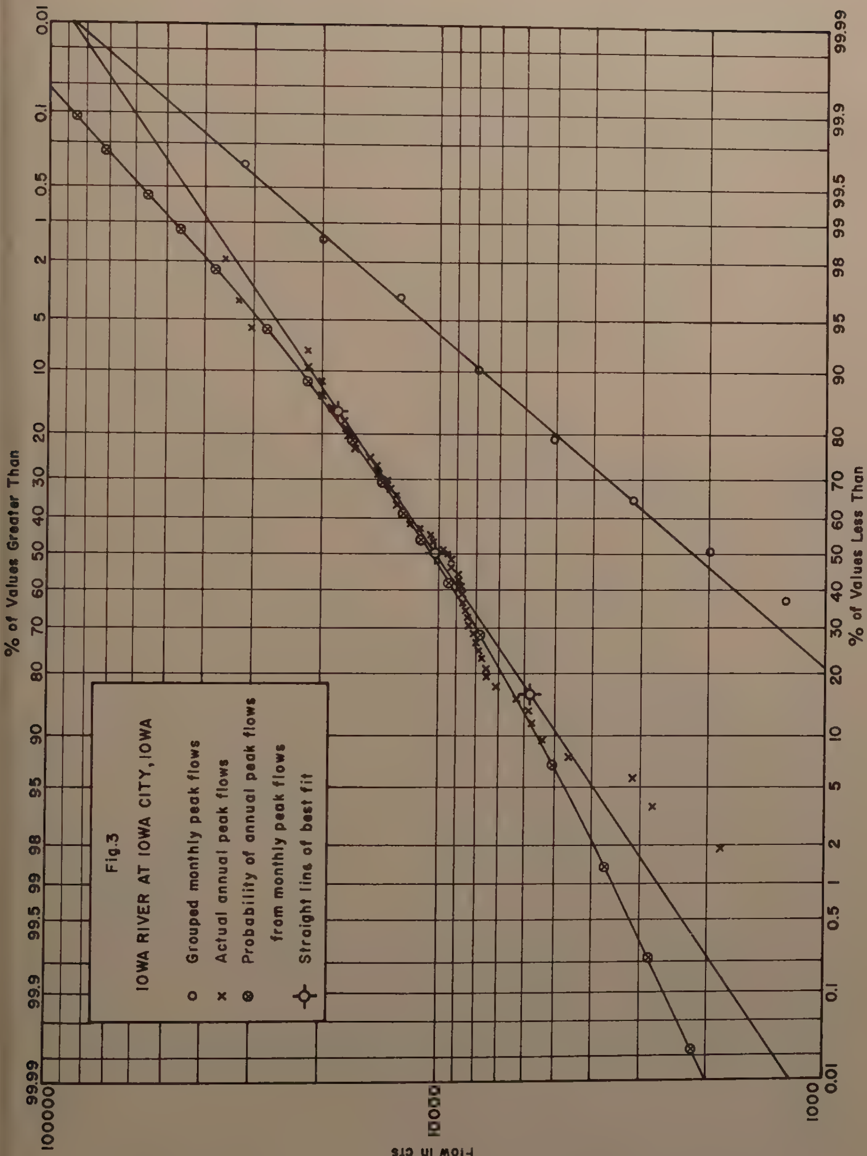
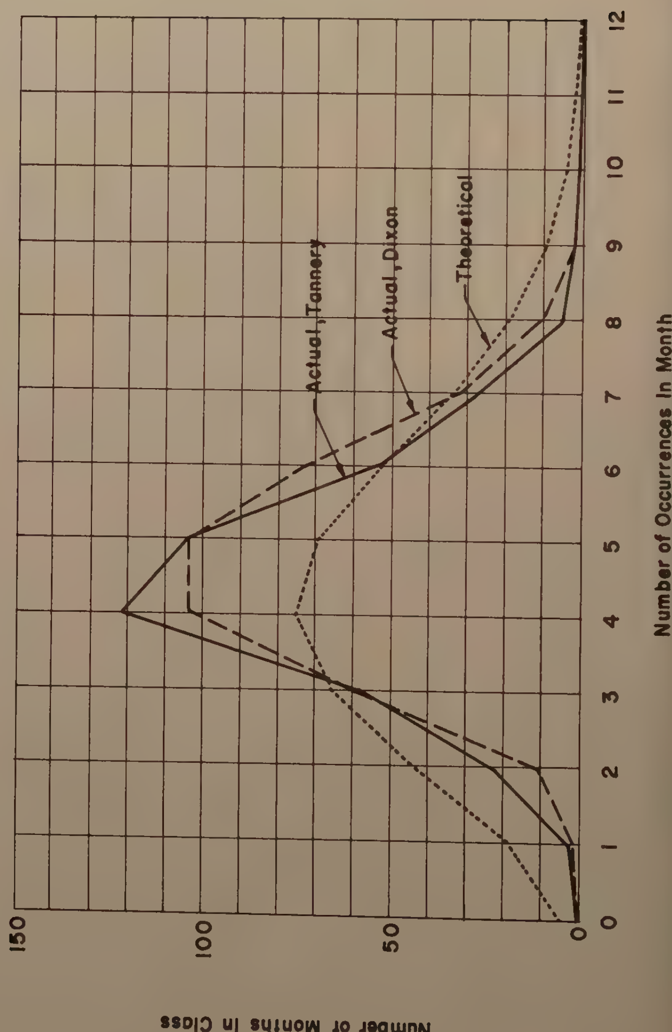


Fig.3
IOWA RIVER AT IOWA CITY, IOWA
○ Grouped monthly peak flows
× Actual annual peak flows
⊗ Probability of annual peak flows from monthly peak flows
Straight line of best fit
% of Values Greater Than
% of Values Less Than
Flow in cfs
100000
10000
1000

Fig.4

NUMBER OF MONTHS WITH VARIOUS PEAK FLOWS



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A HIGH HEAD CAVITATION TEST STAND FOR HYDRAULIC TURBINES[a]

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(Proc. Paper 1201)

SYNOPSIS

This paper presents a description of a new closed-circuit "water tunnel" and indicates its value for testing model hydraulic turbines under high heads (up to 300 feet).

INTRODUCTION

In 1880 the Water Power Company of Holyoke, Massachusetts, built an open flume turbine test stand with a head of 17 feet for testing commercial size water wheels (up to 48" runner diameter). Discharge was measured by weir and computed by the Francis formula which was considered reasonably accurate up to a maximum flow of 225 cubic feet per second. This facility furnished American manufacturers with the opportunity to test and compare the relative efficiencies of their various runner and wheel case designs at reasonable cost.

Beginning in 1915 the hydraulic turbine manufacturers of this country started construction of their own low head open flume laboratories(1) to permit more frequent testing under closer observation of their own engineers, at lower cost and with better protection of confidential design data.

Since the advent of very large hydroelectric units in the 1920's, hydraulic turbine manufacturers have experienced little difficulty in designing and building units capable of attaining 92 to 94 percent peak efficiencies. The major field of exploration in the laboratory, therefore, has been the develop-

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ment of runners with improved cavitation resistance and better operating characteristics.

The first hydraulic turbine cavitation laboratory was built in 1924 by Dr. D. Thoma as an addition to the laboratory of the Hydraulic Institute of the Technical University at Munich, Germany. Dr. Thoma concentrated on improving Kaplan and propeller turbine model designs and it was here that he developed the universally accepted index, sigma, permitting correlation of cavitation data on models and prototypes. The first turbine cavitation laboratory in the United States went into operation in 1930 at the Holtwood plant of the Pennsylvania Water and Power Company, and the various manufacturers followed closely with construction of their own low head cavitation test stands.

A more complete discussion of the function and importance of laboratory investigations in the development of new turbine designs may be found in Reference (2).

General Considerations

The Thoma cavitation index, sigma (σ), may be defined as the ratio of the barometric head (H_b) minus the static suction head (H_s) at any point, usually taken for convenience as the elevation of the blade pivot axes for Kaplan runners and the elevation of the runner discharge for Francis wheels, to the total operating head (H) on the turbine:

$$\sigma = \frac{H_b - H_s}{H}$$

To determine the minimum allowable or critical sigma in the laboratory, the model is usually tested with a constant wicket gate opening (and blade angle in the case of a Kaplan turbine) at a constant speed under a constant head while the elevations of headwater and tailwater are lowered step by step for successive runs until a definite break or change in performance is observed. In practice, a reasonable margin of safety above this break determines the operating limit for the corresponding head and load on the prototype. When a plant sigma drops below the critical sigma for any head and load, cavitation, vibration and loss of output will result.

In this new test stand, medium and high specific speed Francis wheels and all Kaplan and propeller models may be tested under heads equal to those on the prototypes (up to approximately 300 feet). Under these conditions, Thoma's cavitation index is obtained with the same distance between the runner discharge elevation and tailwater as will exist in the field, thereby not only reproducing sigma but actually utilizing the same absolute pressure under the runner as will occur in the prototype. Low specific speed Francis wheels must be tested at less than prototype heads with suction heads somewhat greater than in the full size units, but a much closer correlation between model and prototype is achieved with a high head test stand.

The trend in modern installations toward higher operating speeds and smaller runner diameters,[3,4] while maintaining reasonable settings, makes the ability to test at prototype values of headwater and tailwater especially valuable, since cavitation is the most critical characteristic. In many laboratories, considerable difficulty has been experienced in obtaining definite cavitation break-off points by plotting either efficiency, power output or discharge against sigma values. This is especially true of low specific speed

Francis wheels with their low critical sigma values. Consequently, the chief means of studying cavitation has been by visual and photographic observations utilizing stroboscopic and high-speed high-intensity light sources.

From the very outset, it was decided that the primary function of this test stand would be to obtain accurate and reliable quantitative results on models with water passages homologous to actual prototypes and provisions for visual observation received only minor consideration, since this type of work can be conducted in a low head (50 feet maximum) cavitation test stand which has been in use for many years. The higher test heads are not only more suitable for cavitation research, but the larger horsepower outputs also facilitate a higher degree of accuracy thereby permitting extremely satisfactory tests to be conducted for both performance and cavitation characteristics on the same model.

When running cavitation tests under low heads, in order to obtain low sigma values, it is necessary to utilize large suction heads which frequently cause air to come out of solution and "foul" the manometer lines. By testing under a higher head, the required suction head is smaller for a given sigma value with less suction on the gages and less likelihood of air coming out of solution.

Unusual operating problems can be expected when established limits of practice are disregarded. Laboratory tests duplicating field conditions and permitting investigation of the effects due to their variations are much more likely to lead to trouble-free operation of the prototype units than are tests under much lower heads.

In view of the forgoing technical considerations, supplemented by years of experience with a low head cavitation stand, it was obvious that the new facility would be most valuable if it were designed for the highest practicable head range consistent with reasonable cost. It has previously been agreed that a new turbine test stand was required because of the increasing volume of research and "contract" model testing. No serious consideration was given to adapting the new stand for testing pumps as well as turbines, because of this large volume of turbine work and the fact that a satisfactory pump (and/or pump-turbine) test stand had been in service for some ten years.

Since the new stand would require an extension to the existing laboratory building, of approximately the same size regardless of the test head, and since many of the new equipment components, especially instruments and controls, would be the same in number and approximately equal in price, it was decided that the reasonable increase in total cost for the high head project would be a sound investment.

Test Apparatus

Figure 1 shows the layout of the new test stand. Each of the two centrifugal pumps used to circulate the water delivers 9500 GPM at 118 feet head and is driven by a 300 HP induction motor running at approximately 885 RPM. The pumps are arranged to operate either in series or parallel by appropriate positioning of a Rotovalve and two butterfly valves. Under a head of 300 feet 5000 GPM can be obtained from the two pumps in series, and under a head of 100 feet 21,000 GPM can be obtained from the two pumps in parallel. The head-discharge curves of the pumps are shown in Figure 2. They operate satisfactorily with a suction lift of 20 feet over the entire range

up to the maximum discharge shown and are set 11 feet below the test runner, thereby permitting operation of the pumps without cavitation during any series of tests on a turbine model, even at very low sigma values.

From the pumps, the water flows into the discharge header and then through either one of two Venturi meters, one 30" x 16-1/2" and the other 20" x 10", which were calibrated at the Worcester Polytechnic Institute and the University of Pennsylvanis respectively. Both meters were calibrated in settings similar to their installed locations, using the same straight runs of pipe upstream and the same straightening vanes. Full area Rotovalves are installed in both throat sections for use in the "wide open" or fully closed" positions, and equal lengths of circular pipe were used to simulate them during the calibrations. Selection of the meter for measuring any particular flow is based on the magnitude of the manometer deflections required for accurate readings. Several types of differential manometers, utilizing mercury and water, are available for reading and meter deflections.

From the Venturi meter, the flow is to the intake header and thence to the head tank. The upper section of the vertical riser connecting the intake header to the head tank was designed to provide for an air cushion to minimize any pressure pulsations in the closed system. However, this feature has not been required and the top of the riser merely serves as an air collection and release chamber when starting up the stand. The motor-operated butterfly valve at the bottom of the vertical riser can be adjusted from the control desk to throttle the flow into the head tank and reduce the test head to any desired value.

"Honeycomb" straightening vanes are located in the entrance "neck" of the head tank which flares out to a full seven-foot diameter section where the model spiral and wheel case are mounted.

The size of the head tank is sufficient to permit installation of homologous models of complete prototype spiral cases, inlet valves and special intakes, in addition to the basic turbine (runner and wheel case). All model runners tested to date have had 12 inch throat or discharge diameters, and this size will be adopted for all future tests insofar as practicable. A pheumatically-operated quick-opening door with a bayonet-type seal on the end of the head tank provides easy access to the model under test.

Differential mercury manometers are used to measure net effective heads up to 150 feet. When the 150-foot differential is exceeded, calibrated Bourdon-tube gages are used to measure the inlet pressure head. A mercury manometer is used for measuring the discharge tank pressure.

The entire wheel case, including wicket gates and operating mechanism, stay ring, bottom ring and head cover, is suspended from the dynamometer frame and is mechanically independent of the head tank and other components of the pressurized system. As shown on Figure 3, this is accomplished by attaching the wheel case to a cylindrical barrel, extending through the head tank but connected to it only by means of music note rubber seals. The advantage of this mounting is that any breathing or other movement of the tanks cannot affect the alignment of the turbine shaft which is also attached to the dynamometer and consequently, excessive bearing wear with accompanying extra friction losses and other difficulties are eliminated. A rubber O-ring seal is used between the wheel case and the top draft tube flange in order to complete the independence of the dynomometer and wheel case from the various tanks.

To change the wheel case, the barrel is raised through the top of the head

tank by means of an overhead crane. The wheel case assembly is then removed and placed on the laboratory floor for convenient dismantling and reassembly. The runner can be detached from the shaft and removed from the stand at any time after lowering the elbow portion of the draft tube.

A 300 HP, 0 to 6000 RPM eddy-current dynamometer is used to measure the model horsepower output. The vertical main shaft necessitated a custom-built dynamometer, since only horizontal-shaft units were commercially available, but it was selected to conform with the normal arrangement of prototype units and also because it would simplify accurate torque and thrust measurements. Force is applied at a 21 inch radius to a calibrated horizontal hydraulic load cell and read directly in pounds on the instrument panel. The calibration of the sensitive three-range indicator is checked frequently against a compression-type proving ring periodically calibrated by the U. S. Bureau of Standards. Speed in revolutions per minute is measured either by an electro-mechanical tachometer pulley-driven from the main shaft or by an electronic speed counter with a magnetic pickup actuated by a 60-tooth sprocket on the shaft.

All rotating parts, including turbine runner, shaft and dynamometer rotor, are supported by a Kingsbury thrust bearing in the top of the dynamometer stator which is suspended from a calibrated vertical hydraulic load cell by means of a torsion-free rod. This rod permits convenient determination of axial hydraulic thrust for any test point and also assures that all of the torque developed by the turbine runner will be transferred to the horizontal load cell.

From the head tank, the water flows into the turbine intake and spiral case, through the wheel case and runner, into the top of the draft tube elbow and thence to the five-foot diameter discharge tank. The "bubble" on top of the discharge tank provides for an air cushion in which the discharge pressure can be controlled by adding compressed air and/or water under pressure or by evacuating air with a vacuum pump.

The flow circuit back to the pumps is completed from the discharge tank via the suction header. Total circulation of the 10,000 gallon system capacity would require approximately 30 seconds for a high capacity Kaplan model, increasing to two or three times this figure for a low capacity, high head Francis wheel.

Operation of the stand for an hour could increase the water temperature as much as 15° Fahrenheit. This is controlled (or prevented) by a 100-ton water cooling system. The water to be cooled is taken from the high pressure side of the pumps and returned to the suction side.

City water containing large amounts of air is used to fill the stand after each model change but much of this air is quickly removed by circulation thereby providing water of similar air content to that found at natural hydro sites. Since equipment is available to cool both fresh and de-aerated water, the effect of air content upon performance and cavitation may be studied.

The major indicators and controls (except that for tailwater pressure) have been assembled into an operating console. These include the electronic speed control for the dynamometer, both load cell indicators, electronic and electro-mechanical tachometers, pump motor ammeters and water temperature indicator. Push buttons for remote control of the various valves used for adjusting the head on the model under test or changing from series to parallel operation of the pumps are also located here.

Test Procedures

After the test model is completely assembled in the head tank and the access door has been closed and sealed, the entire system is filled with city water (requiring approximately 15 minutes). The pumps are then started and the entire system is pressurized. All mercury and water manometers are bled and "zeroed." The differential mercury manometers are used to check the calibrations of the Bourdon-tube gages. Test runs are conducted at five-minute intervals, allowing one minute to adjust the new conditions, three minutes for the transients to settle out and one minute for the test itself. Actually, runs could be made conveniently at three-minute intervals by allowing only one minute for the transients to settle, but the five-minute cycle permits the results to be calculated and plotted as the test proceeds.

A complete test on a Francis turbine, including power and efficiency over a range of unit speed (phi), cavitation and runaway speed versus sigma can be completed in four eight-hour days. For a cavitation test, five men are located at the following stations: dynamometer control console, Venturi manometer, headwater manometer, tailwatei manometer and computer's desk. Four of these men double as assemblers for setting up and dismantling the models, thereby acquiring a thorough working knowledge of the test equipment.

During the design stages, the use of measuring equipment that would allow the stand to be operated by one or two men was considered, but it was felt that greater accuracy and more consistent results could be obtained by taking a number of simultaneous readings on all instruments during each run in the same way that prototype field tests are usually conducted. Also, direct manometer reading is more readily understood and allows cross checking by the laboratory technicians, the company's hydraulic engineers, and visiting customers' engineers during "acceptance" tests. Upon completion of a test the stand can be shut down, drained and the runner removed in less than an hour.

Two butterfly valves are used to isolate the head and tail tanks from a loop for testing various types of pipeline valves under high heads and rates of flow.

This new high head test stand is used for basic research and conduct of formal contract model acceptance tests with particular emphasis on duplication of field conditions. A high degree of accuracy is especially desirable because any specific design modification may produce only a small change in performance, which cannot be discovered if it is smaller than the errors of measurement. Since several rather small improvements could add up to a worthwhile increase in efficiency, it is important to be able to identify each one; therefore, instrument errors must be kept as small as practicable.

The results obtained thus far have been extremely gratifying. The original hydraulic and mechanical designs appear to have been quite sound, and only insignificant modifications have been required since it was first placed in operation last year. The repeatable accuracy of any test point appears to be within plus of minus one-quarter of one per cent. The absolute accuracy, as a result of the careful calibration of each component, is considered to be within one per cent.

Test Results

Figure 4 illustrates the type of problem that can be effectively resolved with this equipment. It was desired to increase the capacities of several existing turbines approximately 25 percent, without excessive loss in efficiency, by changing only the runners and wicket gates and continuing to utilize the same inlet valves, spiral cases and draft tubes.

Cavitation was considered to be the greatest obstacle to a successful solution of this problem, since the runner discharge diameter and the plant operating sigma were fixed at their original values by existing conditions. Fortunately, our predecessors had been sufficiently conservative, with normal operating tailwater three feet below the elevation of the runner discharge (sigma = 0.090 at 325 feet head), and it was possible to obtain the specified output of 45,000 HP under 325 feet head and the maximum generator capacity of 50,000 HP under heads of 350 feet and above without encountering signs of general cavitation.

In addition to the new gates and 12-inch discharge diameter runner, the test model included a spiral case, inlet needle valve and several diameters of penstock, all hydraulically homologous to the existing installation, but an existing elbow draft tube model was considered to be suitable for comparison with the existing tubes of the prototype units. These tests were run under a head of approximately 170 feet, with a controlled water temperature of 55° to 60°F. as found in the prototype.

For the curves in Figure 4, output has been "stepped up" to prototype size by the square of the ratio of the runner diameters and by the three-halves power of the ratio of the net heads. The efficiencies are those attained by the model without any increase for the reduced surface friction losses usually attributed to the larger prototype machines.

Figure 5 is an illustration of the accuracy and control attainable in this new stand during one of these cavitation tests, even at very low sigma values. It should be noted that smooth curves are obtained for unit output, discharge and efficiency versus sigma. Critical sigma is indicated as the value corresponding to a loss in output of one percent, well within the output and efficiency margins normally maintained between "expected" and "guaranteed" performance of prototype units.

Prospective customers and hydro plant operators have frequently asked what actually happens in a turbine when the operating sigma drops slightly below the critical value, and whether the output suddenly drops to zero, being replaced entirely by noise and vibration. These questions are answered by Figure 5, specifically for the conditions tested but also generally for any typical operating condition. In this case, a critical sigma of 0.050 is indicated but operation at the very low sigma of 0.007 resulted in a total loss in output of only seven percent and an efficiency drop of only four percent. Obviously, even though objectionable noise and vibration may occur and the resulting cavitation pitting might be serious, unless the affected areas are protected by stainless steel or other resistant material, the residual output is still substantial.

"Emergency" operation at very low sigma values for short periods of time may well be the proper approach in arriving at the most economical setting for any particular turbine.(5) However, for such a study to be effected, a complete and reliable set of model test curves must be available for the various sigma values likely to be encountered.

CLOSURE

By duplicating prototype conditions while maintaining laboratory control and accuracy, homologous model tests can obviate the need for expensive, time-consuming field tests. This is particularly significant when considering Paragraphs 2 and 3 of the ASME Test Code,(6) dealing with inaccuracies of measurement and failure to conform to "ideal" testing conditions.

This new facility is believed to be the most flexible hydraulic turbine test stand in the world today and its availability should benefit the entire industry, including users as well as manufacturers of equipment. As opportunities are presented, not only will accurate and reliable investigations be conducted for specific applications, but basic data will be obtained to demonstrate the effects of various types of test conditions upon performance and cavitation.

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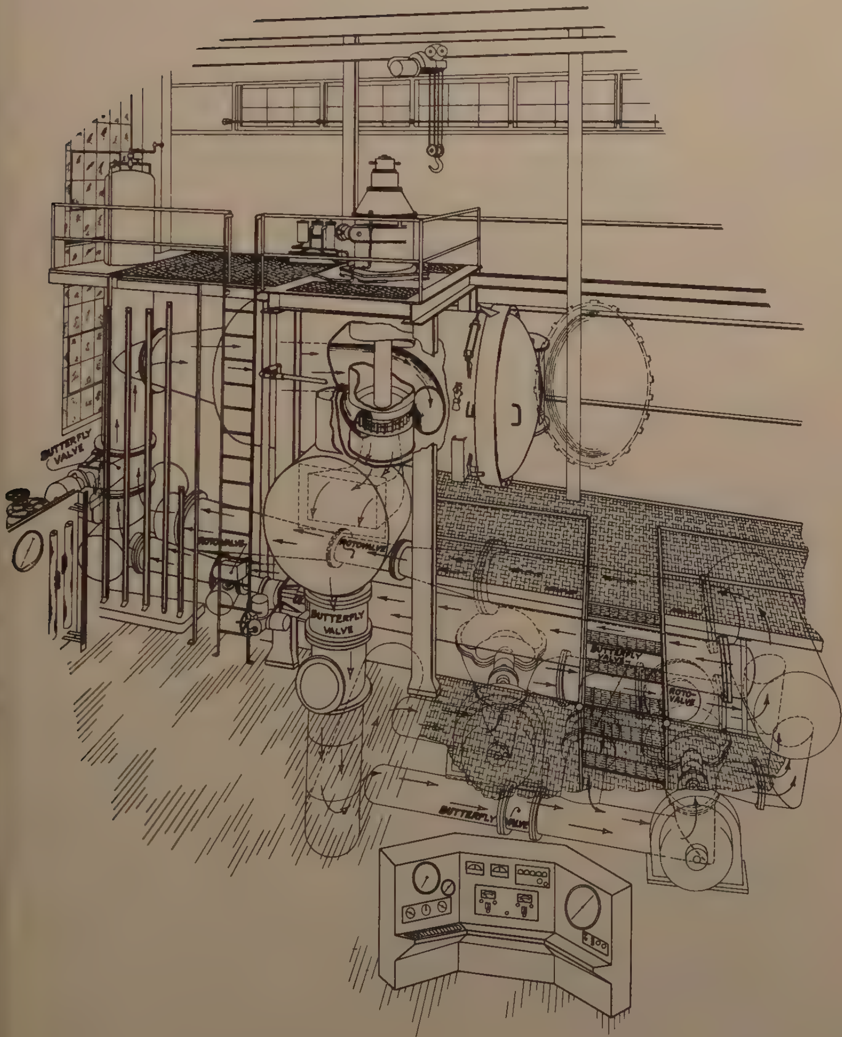
BUTTERFLY VALVE
ROTOVALVE
ROTOVALVE
BUTTERFLY VALVE
BUTTERFLY VALVE
ROTO-VALVE
BUTTERFLY VALVE

FIGURE 1

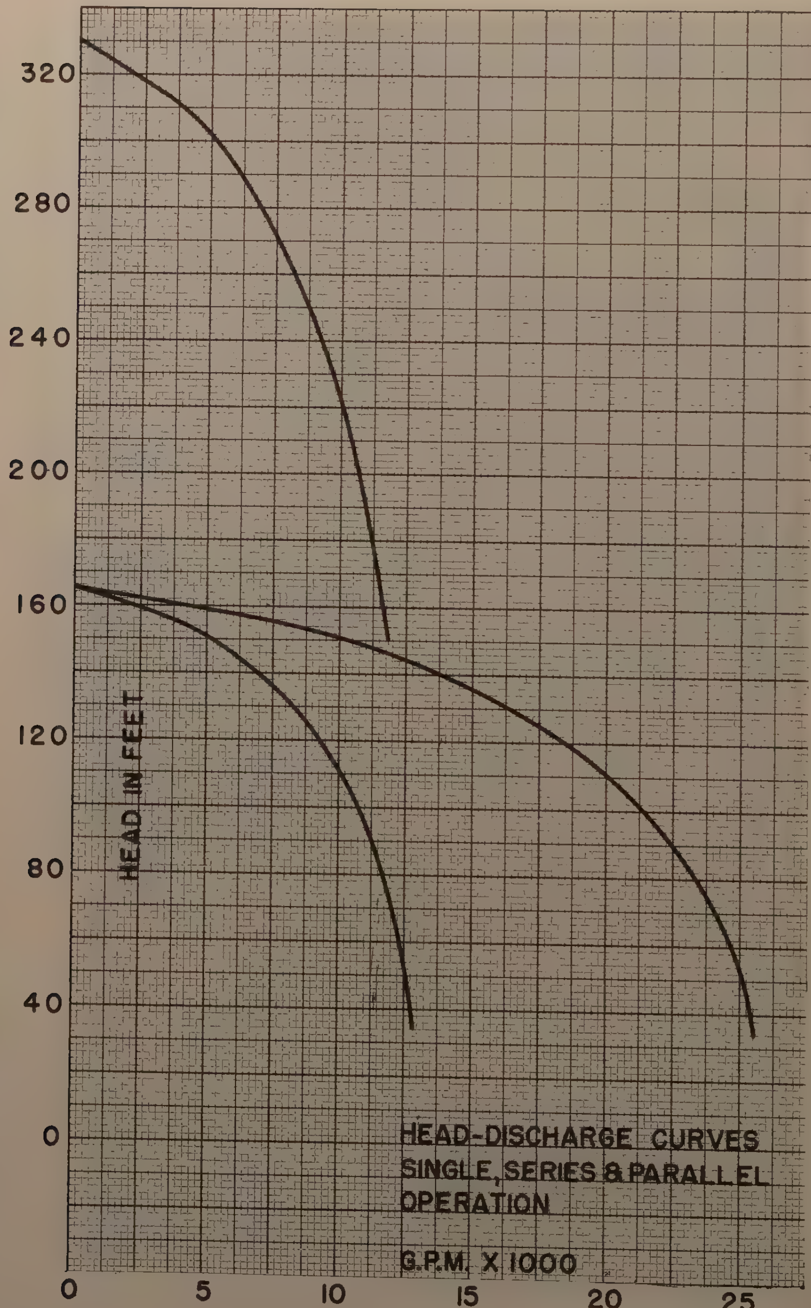
320
280
240
200
160
120
80
40
0
HEAD IN FEET
0
5
10
15
20
25
HEAD-DISCHARGE CURVES
SINGLE, SERIES & PARALLEL
OPERATION
G.P.M. X 1000

FIGURE 2

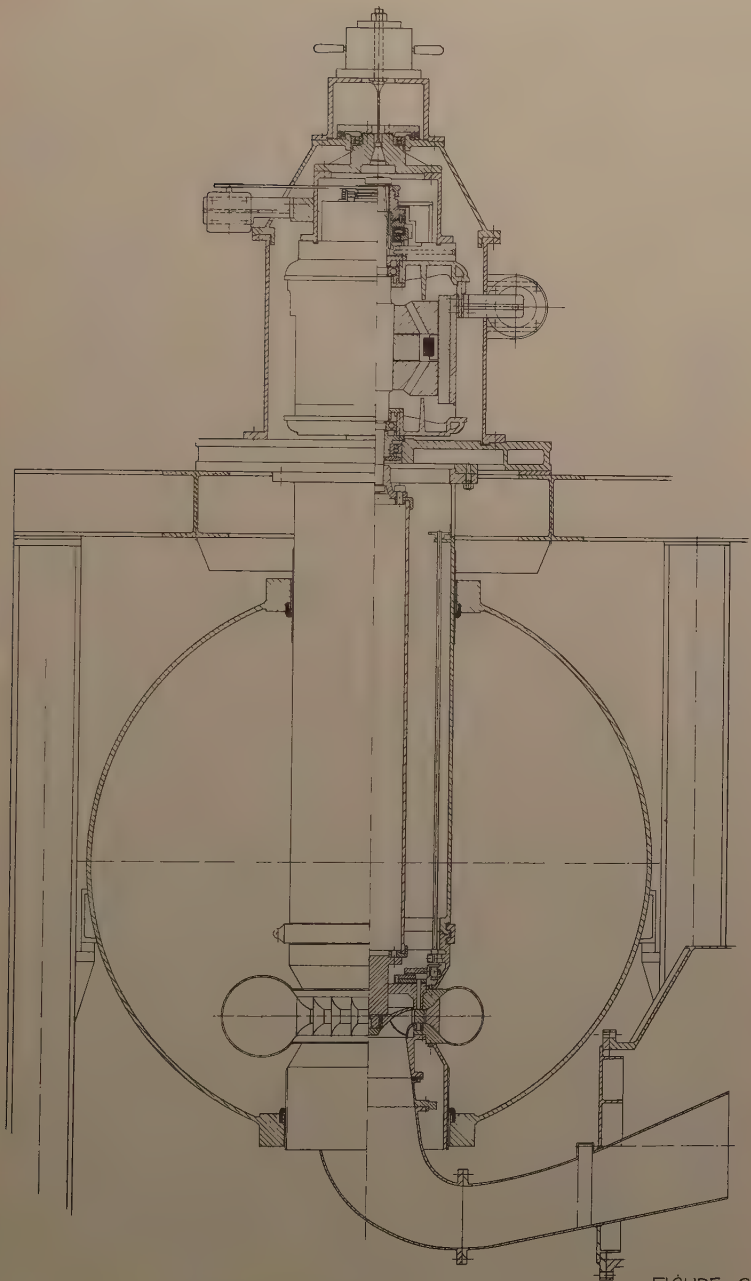

FIGURE 3

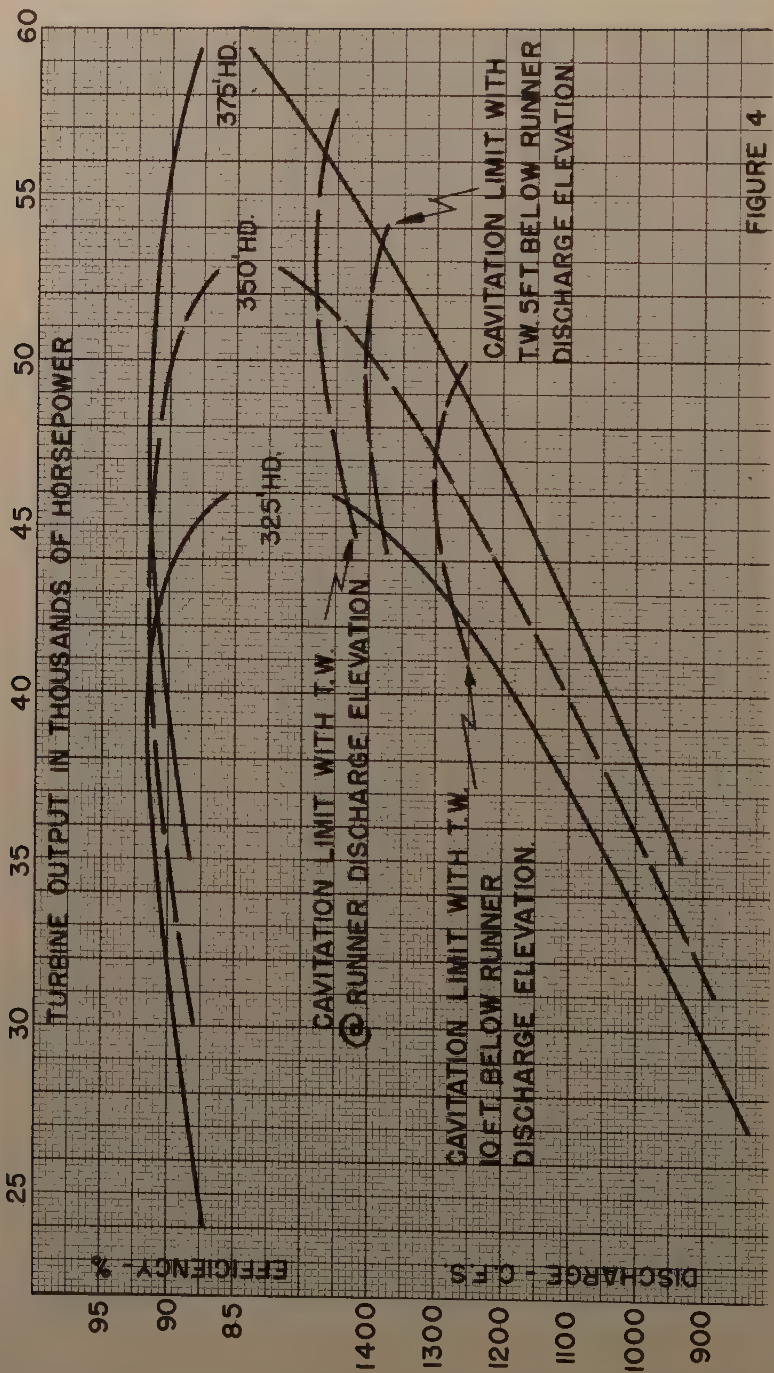
TURBINE OUTPUT IN THOUSANDS OF HORSEPOWER
25
30
35
40
45
50
55
60
EFFICIENCY - %
95
90
85
DISCHARGE - C.F.S.
1400
1300
1200
1100
1000
900
375' HD.
350' HD.
325' HD.
CAVITATION LIMIT WITH T.W. @ RUNNER DISCHARGE ELEVATION
CAVITATION LIMIT WITH T.W. 10 FT. BELOW RUNNER DISCHARGE ELEVATION.
CAVITATION LIMIT WITH T.W. 5 FT. BELOW RUNNER DISCHARGE ELEVATION.

FIGURE 4

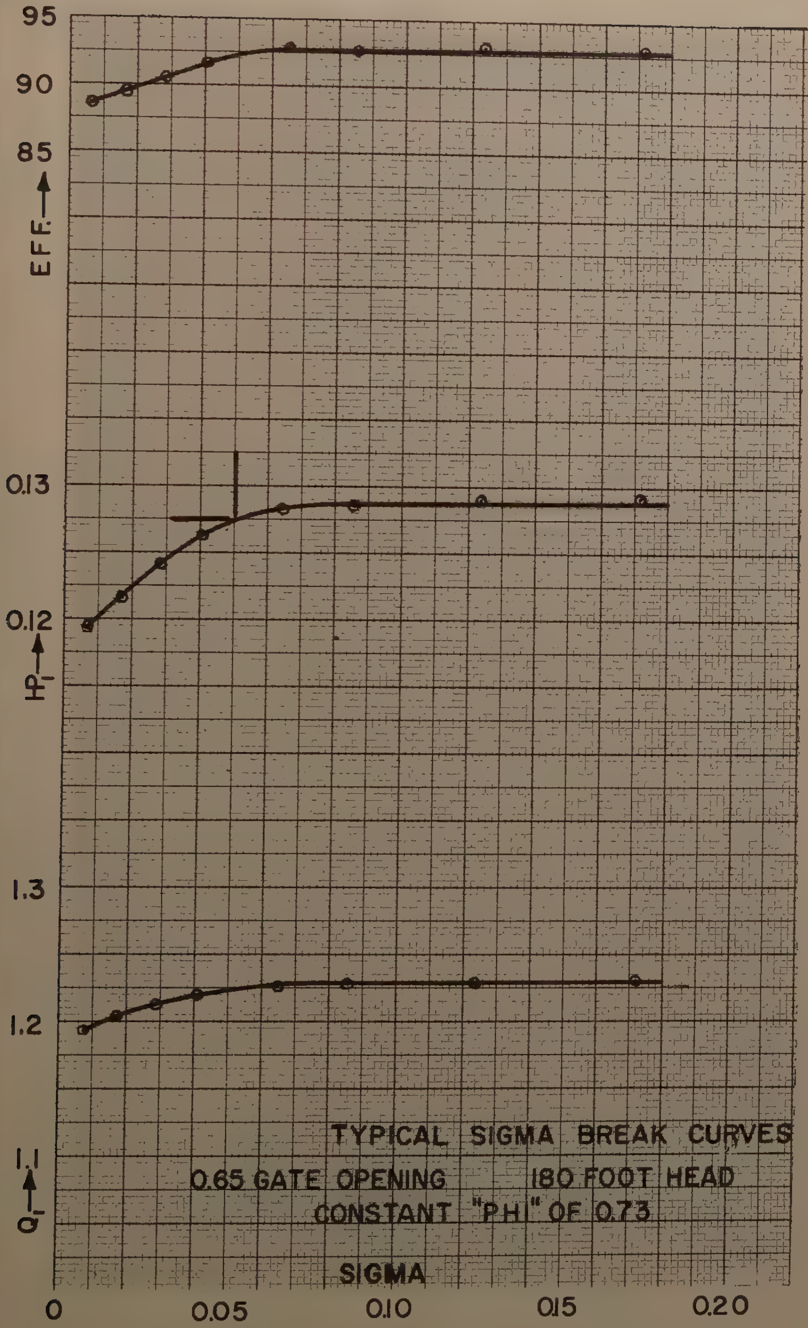
95
90
85
EFF.
0.13
0.12
HP1
1.3
1.2
1.1
Q1
TYPICAL SIGMA BREAK CURVES
0.65 GATE OPENING 180 FOOT HEAD
CONSTANT "PHI" OF 0.73
SIGMA
0
0.05
0.10
0.15
0.20

FIGURE-5

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IS THE WRITING OF FLOOD INSURANCE FEASIBLE?

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(Proc. Paper 1202)

FOREWORD

This is the third and last of a series of papers on Flood Insurance presented at a session of the Hydraulics Division during the Annual Convention of the Society, Pittsburgh, Pa., October 17, 1956. The first two papers of the group appeared as Proc. Papers 1164 and 1165 in the February 1957 Journal of the Hydraulics Division, as follows:

Proc. Paper 1164 "Some Physical Problems Related to Flood Insurance" by A. Arthur Koch.

Proc. Paper 1165 "Technical Problems of Flood Insurance" by H. Alden Foster.

The purpose of this paper is to state the position of the Capital Stock Insurance Industry on the general subject of flood insurance from an underwriting standpoint. Accordingly, the credentials and extent of the representation of the writer should be informally presented, keeping in mind that this is a subject which is broad enough and of the fluid character which permits opposing views.

The American Insurance Association, which the writer represents and whose views are expressed, embraces within its membership 195 domestic and alien stock insurance companies licensed and admitted to do the business of fire, marine, and casualty in the United States, its Territories, and possessions. Most members transact a world-wide business. The Association property insurance companies transact probably more than 80 per cent of the total insurance written by stock insurers in the United States and probably more than 65 to 70 per cent of the total property insurance business transacted by all admitted property insurance companies, stock, mutual, and reciprocal.

Note: Discussion open until September 1, 1957. Paper 1202 is part of the copyrighted Journal of the Hydraulics Division of the American Society of Civil Engineers, Vol. 83, No. HY 2, April, 1957.

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Traditionally careful underwriters have taken the view that specific flood insurance on fixed location properties cannot be successfully written. By that is meant that such an endeavor is not one which insurance companies could enter into without unduly jeopardizing their solvency and without casting some doubt on their ability to respond to their outstanding commitments as regards the many other perils which are written. While this position was generally accepted throughout the industry on the basis of educated underwriting judgment and experience, in 1944 the companies formed a representative committee of the industry to make a further study of the subject. Opinions and conclusions of competent engineers were secured in this connection and the result buttressed the belief that specific flood coverage could not be written in areas subject to recurrent floods on an insurance basis.

Following the terrible damage resulting from the floods of May and July of 1951 in Kansas and Missouri, insurance underwriters embarked again upon a re-examination and re-evaluation of their traditional position. Naturally, a catastrophe of this magnitude aroused public interest anew in the subject of flood cover and for this, as well as other reasons, such a study was undertaken with the firm knowledge and belief that following such a course was a good business practice. This study was completed with the assistance of the engineering firm of Parsons, Brinckerhoff, Hall & Macdonald and the results were given wide distribution. Again, the original position of the underwriters was sustained.

In 1955, following the August floods in the Northeastern states, the Capital Stock Insurance business asked the same engineering firm to study the technical engineering aspects of the subject of floods and flood damage and bring their previous study up to date. While this work was proceeding, the October 1955 floods struck in the same general area and the engineers were requested to enlarge their work to include this phenomenon. In December 1955 the West Coast floods occurred, hitting California especially hard, and the engineers again expanded the territorial scope of their work to include the damage resulting from the West Coast floods.

On January 10, 1956, American Insurance Association adopted a comprehensive statement of the position of the Capital Stock Insurance business on the subject of floods and flood damage. While this statement was released prior to the completion of all the engineering work on the various floods occurring in 1955-1956, a subsequent review of the feasibility of Stock Insurance companies providing flood insurance has resulted in a strengthening of the conclusion that such coverage cannot be successfully written as a commercial insurance venture. As a matter of fact, nothing in the engineering reports or in the further study of the insurance aspects of this subject has provided any basis for altering the traditional position of the Stock Insurance business.

American Insurance Association, through its Committee on Floods and Flood Damage, has recently published a book titled "Studies of Floods and Flood Damage 1952-1955" which includes all the pertinent material developed by the industry during that period as well as the comprehensive reports of the engineering aspects of the problem prepared for the industry. The record of efforts of the industry to come to a valid conclusion in connection with the flood problem is set forth in this book in full.

The 1952 report of the engineers clearly demonstrates that on a purely theoretical basis specific flood insurance could be rated and written if certain fundamental requirements of insurance could be met. The applica-

tion of the engineers' findings to the practical underwriting considerations of the insurance business makes it obvious that the fundamental requirements of insurance could not be met in a specific flood insurance undertaking. The considerations which dictated this conclusion will be discussed briefly.

It would be misleading to leave the impression that insurance companies are presently free of exposure to loss from flood. To the contrary, there are flood loss potentials of catastrophic proportions under various forms of existing insurance coverages. For example, practically all motor vehicles are insured under a comprehensive coverage which includes damage resulting from flood. Then, too, so-called Inland Marine insurance policies insuring merchandise in transit or on consignment usually include flood coverage. This is equally true of certain types of so-called All Risk policies which insure bridges, tunnels, dredges, contractor's equipment, jewelry, furs, objects of art, certain personal property, and so forth. Marine insurance policies insuring yachts, tugs and equipment, cargo, merchandise in transit or in storage on piers and in warehouses are not infrequently called upon to pay for loss or damage resulting from flood tides or high water. It should be well noted, however, that, with the exception of policies covering bridges, tunnels, and fixed property of a similar nature, virtually all of these present forms of insurance which include flood coverage have one thing in common, i.e., they cover property mobile in character and which presumably has a chance of being moved out of danger when a flood threatens.

Nor should the impression be left that the business of insurance is unduly fearful of catastrophes which might occur within the scope of outstanding insurance policies. While everything prudently possible is done to minimize the probability of loss of life and property, the business is eager to discharge equitably and promptly all just claims when they do occur. The Texas City explosion of 1947 cost casualty and property insurance companies $85,000,000 which was met in the regular course of business. The very nature of insurance in action is surprise, the unexpected. While private insurers are able profitably to cover risks of other disasters where catastrophic losses occur relatively infrequently or comparatively minor losses occur frequently, they do not believe it feasible to attempt to insure such risks as flood and high water when losses not only are catastrophic but also occur frequently.

The foregoing should demonstrate that the companies would write flood insurance if it were thought they could consistent with good business practice. As a matter of fact, the companies would be not only willing but eager to provide it.

One of the most baffling aspects of the flood insurance problem is to develop a reasonable definition of the word "flood." It is defined in a variety of ways, and generally very loosely. For insurance purposes, it would be difficult to support and maintain a narrow or restricted definition of flood, yet a broad definition that would include any kind of inundation would present a variety of complications, one of which is that many properties subject to almost certain periodic loss would be offered for insurance coverage. If such properties were attempted to be covered, the rates would necessarily be so high as to deny flood coverage to those risks which need it most. In this connection it can be observed that any insurance program which denied coverage to those risks which need it most and which were probably subject to recurring floods would not meet the public need.

Even if specific flood insurance could be offered at a reasonable premium,

experience has indicated that only those who have property exposed to the possibility of damage by flood would buy it. This is called adverse selection. As a result, the spread of risks essential to sound insurance underwriting would be lacking since all properties insured would be known to be subject to periodic loss by flood. It can truly be said that each flood would produce a catastrophe loss, comparatively, in that all properties, subject to damage by a flood of given intensity, would actually be damaged by such a flood, except property which could readily be removed from the affected area.

Some years ago several companies seriously attempted to write specific flood insurance only to learn that there was a very limited demand for the coverage, that the peak demand comes directly after a flood, and that property owners generally did not maintain their flood insurance in force after a few floodless years. It was also learned that because of adverse selection it was impossible to get the necessary spread of risk, with the result that the companies were obliged to charge rates commensurate to the exposure, which property owners could not or would not pay.

If a program of flood indemnity were otherwise feasible, the expense involved in the individual rating of risks would probably be prohibitive if the coverage were to be offered on an actuarial, sound basis and without unfair discrimination. Not only a complete hydrological survey of each river basin and flood area, but also a detailed hydrological survey of each "reach" of each river would be necessary. In addition to the above, a detailed survey and appraisal of each property to be insured would have to be obtained.

In their studies, the engineers outlined a method for estimating maximum probable flood loss for an individual property as well as the average annual loss for an individual property. The average annual loss is the amount which an insurer would have to charge merely to pay losses, and to this there would have to be added a loading to cover expenses. Thus, the average annual loss for an individual property plus the expense loading would represent the premium which an insurer would have to charge the property owner for protection equal to his estimated maximum probable loss.

Using the methods outlined by them, the engineers computed average annual losses for certain properties in several flood areas. These computations show conclusively that flood insurance on those properties on which such coverage is most needed could be offered only at a prohibitive premium because of the virtually certain loss and therefore, for practical purposes, these properties would be uninsurable.

The problem of an equitable distribution of the cost of a flood insurance program presents insurmountable obstacles. Fair dealing dictates that the cost of flood insurance should be borne by those whose properties are exposed to the peril of flood and who would voluntarily choose to purchase specific flood insurance. To attempt to distribute the cost of such a program among all insureds regardless of their exposure to the peril of flood would be manifestly unfair and inequitable. It somewhat outrages common sense to expect that those property owners without flood exposure would agree to bear a part of the cost. And any proposed plan that involves charging a fee to a person not subject to one or more of the perils covered by the policy is inconsistent with sound insurance principles and incompatible with the idea of a free economy.

However, some have seriously urged that the cost of flood insurance be distributed as a loading on Extended Coverage rates and that the peril of flood be included in that coverage. The companies believe that such a

course is impracticable because, unlike other natural catastrophes which are unpredictable as to place of occurrence, floods can occur only where water flows or gathers. Only those properties which are in the path of the flow of water or gathering have any need for it. Competition would then force the sale of coverage without flood as to properties considered immune. It follows from this that only those buyers who have a flood exposure would purchase the flood cover, thus engaging in the practice of adverse selection.

To be more specific, any insurer adding flood coverage to those perils now covered under the Extended Coverage Endorsement may lose business to its competitor who does not follow such a practice. The result is almost inevitable because the loading of the rate to cover the peril of flood would be substantial in amount. It is estimated that rates for Extended Coverage insurance would have to be doubled at least, assuming that all present Extended Coverage insureds would purchase the flood cover.

As briefly adverted to earlier in this paper, the continued solvency of insurance companies is a compelling factor in the considered decision that specific flood insurance cannot be written feasibly. Even if practical considerations could be disregarded and particularly if it could be assumed that property owners with a flood exposure would buy flood insurance at rates established according to sound actuarial principles, the catastrophe potential of such an undertaking is so great as to threaten the solvency of the entire property insurance business. It is doubtful whether the aggregate net free assets of all insurers would be adequate to withstand the constant drain of the recurrent catastrophe losses that would be inherent in flood insurance.

If it could be established that the risk of loss by flood is a proper subject for insurance, that specific flood insurance could be written on a sound basis, and that the public would purchase specific flood insurance at necessary rate levels, it is obvious that companies generally would desire to engage in this field. Because of the virtual certainty of the loss, its catastrophic nature, and the impossibility of making this line of insurance self-supporting due to refusal of the public to purchase such insurance at the rates which would have to be charged to pay annual losses, companies generally could not prudently engage in this field of underwriting. Accordingly, it is the considered opinion of the companies that insurance against the peril of flood applicable to fixed property cannot successfully be written.

The position of the Stock property insurance companies, as herein stated, has been widely known, certainly since the First Session of the 82nd Congress when the Committee on Appropriations of the House of Representatives held hearings on the request for appropriations for flood disaster relief and rehabilitation as a result of the Kansas-Missouri floods of 1951. Contained in the general appropriation request was a proposed plan to indemnify flood victims for physical loss of or damage to tangible real or personal property up to 80% of the amount of the loss, provided that the amount to be paid any one person submitting a claim did not exceed $20,000.

Representatives of Capital Stock Insurance business testified at these hearings, at the request of the Committee, giving testimony on the technical aspects of the indemnity program then under consideration. The proposed indemnity program for flood victims was not favorably considered by Congress at that time. Government programs were then restricted to the granting of relief to flood victims and assisting in their rehabilitation.

Following the devastating floods of 1955 and 1956 in the Northeastern and Western parts of the United States, several bills providing for so-called

flood insurance were introduced in both the House of Representatives and the Senate. Extensive hearings were held on the proposal to provide flood indemnity by a program under the aegis of the Federal Government. Representatives of the Capital Stock Insurance business again made themselves available to give testimony and otherwise to assist the Government in the study it was undertaking. The conclusion of the Stock Insurance business that a program of specific flood insurance was infeasible was stated and the reasons upon which this conclusion was based were given in detail to the Congressional Committees.

In addition, insurance company representatives pointed out that:

1. The companies believe that the Government would encounter the same obstacles if it undertook a program of specific flood indemnity by means of insurance on a self-sustaining basis.

2. It was further suggested that any Government promise of indemnity on a non-self-sustaining basis is "relief" in the guise of insurance. It was the position of the companies that a direct program of relief and rehabilitation would be more effective and more equitable, particularly in restoring essential services and providing food and shelter, which are the first forms of necessary relief in the case of a major flood disaster.

3. In the opinion of the industry, flood control and prevention (rather than insurance, indemnity, or relief) are of far greater importance to potential flood victims, especially when the many forms of irremediable losses are also taken into consideration, such as death, bodily injury, loss or employment and loss of income.

The companies, however, made it abundantly clear—and very sincerely—that if Congress, in its wisdom, deemed it in the best interests of the people of the United States to adopt a program of flood indemnity, the Stock Insurance industry would cooperate to the fullest extent by making their full facilities available to the Government. The producers also offered their services.

The Congress adopted the "Federal Flood Insurance Act of 1956," which was signed into law by President Eisenhower on August 7, 1956. This law, also known as Public Law 1016—84th Congress, 2nd Session—according to the President's statement, "directs the Housing & Home Finance Administrator to establish a system of indemnification, within limits, for losses sustained in flood and tidal disaster; to re-insure private insurance coverage of such losses; and to assure a line of credit, where necessary, for the restoration and reconstruction of properties damaged or lost as a result of flood."

This experimental program, according to the President, does not propose putting the Federal Government permanently into the flood insurance business. The hope is expressed that the Government can lead the way on a basis that will enable private endeavor to occupy this field in the shortest possible time. The full cooperation and active support of the private insurance carriers is declared to be essential to the successful accomplishment of the law's immediate and ultimate objectives.

The law provides for a maximum public subsidy of 40% to supplement fee payment by those who apply for and are issued flood indemnity, with State participation in the subsidy being deferred until July 1, 1959.

Consistent with its sincere offer, previously made, to assist in any way it

could upon receipt of the Government's request, the Insurance industry is presently engaged in giving counsel and other forms of assistance to the Administrator charged with the responsibility for the program under the law. The industry meant what it said when it offered assistance and its full facilities are being presently made available in an honest and sincere effort to make the program a success.

Journal of the
HYDRAULICS DIVISION
Proceedings of the American Society of Civil Engineers

CONTENTS

DISCUSSION
(Proc. Paper 1230)

(Over)

*There will be no closing discussion to this paper.

Discussion of
"MECHANICS OF STREAMS WITH MOVABLE BEDS OF FINE SAND"

by Normal H. Brooks
(Proc. Paper 668)

NORMAN H. BROOKS,[1] J.M. ASCE.—The writer certainly welcomes all the vigorous discussion that has been presented for this paper. The critical nature of all these discussions has certainly demonstrated that the writer's findings and ideas are not generally recognized and accepted by the engineering profession in general. In this closing discussion the writer will attempt to answer the various criticisms of the discussers.

INTRODUCTION

In presenting this paper the writer realized that the scope of the experimental investigation was far too limited for the purpose of deriving general quantitative relationships between the various characteristics of alluvial streams which carry an appreciable amount of bed material in suspension. The data were presented in as simple a form as possible and no attempt was made at any kind of numerical or dimensional analysis. In fact, it was felt that it would be helpful for the profession to examine the basic data to see what some of the general interrelationships are. Before dimensional analysis can be successfully applied to any problem, one should know which variables are pertinent and what functional relationships it would be most worthwhile to seek.

The writer does not claim to have discovered that dunes affect channel roughness, as this has been realized for a long time. However, the writer disagrees with the generally accepted assumption that the total roughness can be uniquely determined from the grain size distribution of the bed material, the water cross section, and the slope (or the bed shear). For example, even though Einstein and Barbarossa (14)[2] separate the bed shear into two parts, one for the actual grain roughness and the other for the roughness of the dunes or bars, they still imply that the friction factor and discharge can all be determined uniquely from the variables mentioned above.

Several of the discussers seem to have had difficulty in grasping the meaning of uniqueness or single-valuedness of a functional relation. As an almost trivial example, consider the simple relation $y = \sin x$. Now y is certainly a unique function of x because for any value of x there is only one possible value of y. However, if we write $x = \sin^{-1} y$ we find that the value of x is not uniquely determined by the value of y. Similarly then, the observation that the slope is a unique function of depth and velocity (or discharge) does not mean that velocity must, a priori, be a unique function of depth and slope. Just as in the example above, it may turn out that several combinations of

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2. Numbers in parenthesis refer to references of original paper (No. 668, April, 1955).

depth and velocity yield identical values of the slope. In this case, if we start our consideration with a given depth and slope, it is certainly reasonable, from a logical point of view, to expect that there may be more than one possible velocity. Needless to say, in analysis of problems in alluvial hydraulics it would be helpful to have relationships which are unique so that one will not be troubled with a multiplicity of solutions. It is on this basis that the writer has suggested that one should avoid thinking of depth and slope as independent variables because of the possible multiple-valued nature of the relationships which result.

The organization of the following discussion roughly parallels that of the original paper. Questions raised in regard to apparatus and procedure will be answered first; following that, some new data which are pertinent to the discussion will be presented. The main part of this closure will be concerned with the discussers' interpretations of the laboratory data. Finally the discussion will be concluded with points relating to field observations. Arguments advanced by the different discussers will be considered in the appropriate sections of the discussion below.

Apparatus and Procedure

Professor Blench, and Messrs. Carlson and Mostafa have raised doubts that true equilibrium for a run had been established in the flume by the time the data were taken. During runs, a continuous check of the stability was easily made by taking water-surface profiles frequently. After the initial period required to reach equilibrium, which sometimes was as short as one-half hour, the water surface elevations never changed by any significant amount. Furthermore, complete water-surface and bed-surface profiles taken after running different lengths of time showed identical results. This then, would rule out the possibility suggested by Professor Blench that, "the same conditions applied for the same time produce the same degree of disequilibrium." The writer is confident that good stable equilibrium for both the water and sediment discharge was achieved for each run with the possible exceptions of Runs 12 and 24, for which there were long flat sand waves (i.e., both flat and dune-covered sections occurred simultaneously in the flume).

Messrs. Carlson and Mostafa also question the regulation of the depth of flow during the establishment of a run. Since all the experiments were performed in a closed-circuit flume as shown in Fig. 1, the average depth is governed solely by the amount of water put into the system. Thus, from the very beginning of a run the depth is necessarily maintained at a correct average value, although there is a possibility for a slight transient non-uniformity.

On page 841-31 Dr. Liu questions the uniformity of flow conditions represented by Fig. 2 for Run 27. By inspection of the figure it is found that the maximum variation of the depth is $\pm$ .003 ft with an average value of 0.231 ft. In the bed elevation the deviations are +.003, -.002 ft (after leveling in four-foot reaches), and the fluctuations in the water-surface readings are even less. The writer believes that these variations are exceedingly small as flume experiments go, and that the large number of water-surface readings taken for each run should guarantee that the deviations are no larger. Furthermore, since the discusser's Eq. (a) for the shear ($\tau = \gamma rS$) has been used almost universally by hydraulic engineers for cases of gradually

converging or diverging flow in developing what are commonly called backwater and drawdown curves, the writer can find no basis for the discussers' criticism that Eq. (a) should not be used.

In regard to a further question by Dr. Liu about the procedure for stopping the flume, it may be stated that the pump was abruptly stopped. At this instant a large wave was formed at the downstream end, and traveled upstream to the inlet. The passage of this large wave was observed to displace only a relatively few grains of sand, mostly knocking them off the dune crests into the troughs. Hence, the effect of the disturbance due to the stopping of the flow is considered to be nil as far as the mean depth of the sand bed is concerned. It should be remembered that the system is closed so that there is no drainage of water off the bed at the time that a run is stopped.

When the flow is stopped the sediment in suspension settles to the bed, thereby necessitating a correction in the total depth as pointed out by Dr. Liu. The depth of deposition of the suspended sediment was computed and found to be less than .001 ft in all but two or three cases for which corrections of .001 ft were made.

The writer concurs with Messrs. Carlson and Mostafa, and Dr. Liu in the opinion that it would have been advisable to use more sand in the flume. Under some conditions small bare spots did appear on the bottom. However, had there been more sand in the flume, the roughness for some of the runs with dunes might have been even higher, tending thereby to enforce the writer's conclusions rather than to weaken them. In subsequent experiments, more sand has been used; yet these experiments lead to the same general conclusions.

Dr. Barton has raised a valid criticism of the writer's use of the term "smooth." Since smooth already has a special meaning in hydrodynamics, the use of the word "plane" or "flat" would be better to describe the condition of the bed in the absence of dunes. The writer prefers the term "flat" because it does not imply the preciseness of the word "plane", and is already commonly used in describing natural topography of a large scale.

New Experimental Results

The experiments runs reported in the original paper were performed during 1953-54 as part of the thesis work done by the writer for the Ph.D. degree at the California Institute of Technology. Since then, further investigations along these same lines have been carried out by Dr. Vito A. Vanoni and the writer under sponsorship of the Corps of Engineers, Omaha Division, and the National Science Foundation. The results of this research which are pertinent to this discussion are presented in Tables 7 and 8.

The scope of the original investigation has been expanded in essentially two ways: (1) by using sands with a wider gradation of grain sizes, and (2) by making similar experiments in a wider and longer flume. The distribution of sieve sizes for the three new sands used, designated Nos. 3, 4, and 5 respectively, are shown graphically in Fig. 26. The characteristics of the size distribution of all five sands are summarized in Table 9. Although the geometric mean sieve diameters (D_g) for sands No. 3, 4 and 5 are very close to that for sand No. 1, the geometric standard deviations (σ_g) are considerably larger. Values of the mean sedimentation diameter are presented in Table 9 only for sands 1 and 2 where σ_g is quite small. For the other sands the

TABLE 7

SUMMARY OF EXPERIMENTS BY NOMICOS IN 10.5-INCH FLUME, 1956

Run No.	Q Dis-charge cfs	d Depth ft	r Hydr. Radius ft	S Slope	U_* Shear Vel. ft/sec	U Ave. Vel. ft/sec	f Frict. Factor	T Water Temp. °C	r_b Bed Hydr. Radius ft	U_{*b} Bed Shear Vel. ft/sec	f_b Bed Frict. Factor	No. of Sed. Disch Samples	$\bar{C}$ Sed. Disch. Conc. gr/l	G Sed. Dis-charge lb/min	Analysis of Sediment Load D_g mm	Analysis of Sediment Load σ_g	F Froude No.	Bed Condition	Run No.
Series I, Sand No. 3, D_g = 0.145 mm, σ_g = 1.30																			
A	0.435	0.241	0.156	0.0021	0.103	2.06	0.020	25.0	0.166	0.106	0.021	9	1.85	3.0	-	-	0.74	Flat	A
B	0.269	0.242	0.156	0.0027	0.116	1.27	0.068	25.0	0.187	0.136	0.092	10	1.2	1.2	-	-	0.45	Dunes	B
C	0.193	0.241	0.156	0.0021	0.103	0.91	0.101	25.0	0.218	0.121	0.141	5	0.23	0.17	-	-	0.33	Dunes	C
Series III, Sand No. 4, D_g = 0.137 mm, σ_g = 1.38																			
H-2	0.436	0.233	0.152	0.0025	0.111	2.13	0.0215	24.3	0.167	0.116	0.0235	7	2.3	3.8	0.155	1.35	0.78	Flat	H-2
H-3	0.387	0.223	0.148	0.00225	0.104	1.97	0.022	24.0	0.162	0.108	0.024	12	3.3	4.7	0.096	1.60	0.73	Flat(s.w)*	H-3
H-7	0.293	0.237	0.154	0.00275	0.117	1.41	0.054	24.1	0.199	0.133	0.071	12	2.0	2.2	0.093	1.56	0.51	Sand Wave#	H-7
H-7a	0.293	0.243	0.156	0.00275	0.118	1.38	0.059	24.6	0.210	0.136	0.078	-	-	-	-	-	0.49	Dunes	H-7a
Series II, Sand No. 5, D_g = 0.152 mm, σ_g = 1.76																			
2b	0.170	0.241	0.156	0.0020	0.100	0.80	0.124	26.0	0.220	0.119	0.175	6	0.30	0.19	0.070	1.7	0.29	Dunes	2b
2a	0.180	0.241	0.156	0.0021	0.102	0.85	0.115	25.6	0.219	0.122	0.162	6	0.59	0.40	0.072	1.9	0.31	Dunes	2a
2	0.193	0.241	0.156	0.0024	0.110	0.91	0.115	25.5	0.220	0.130	0.163	12	0.82	0.59	0.069	2.1	0.33	Dunes	2
3a	0.207	0.241	0.156	0.0026	0.114	0.98	0.108	25.0	0.219	0.136	0.153	7	1.15	0.88	0.065	1.7	0.35	Dunes	3a
3	0.219	0.241	0.156	0.00275	0.118	1.04	0.103	25.0	0.220	0.140	0.145	7	1.8	1.5	0.061	1.9	0.37	Dunes	3
4	0.253	0.241	0.156	0.0027	0.116	1.20	0.075	25.0	0.214	0.136	0.103	7	2.5	2.4	0.062	1.6	0.43	Dunes	4
5	0.292	0.241	0.156	0.0024	0.109	1.38	0.050	25.0	0.203	0.125	0.065	14	3.4	3.8	0.064	1.7	0.50	Dunes	5
6	0.327	0.241	0.156	0.00225	0.106	1.55	0.038	25.0	0.195	0.119	0.047	14	2.9	3.5	0.065	1.7	0.55	Sand Wave#	6
7	0.358	0.241	0.156	0.0021	0.102	1.69	0.029	25.0	0.185	0.112	0.035	14	3.3	4.4	0.071	1.8	0.61	Sand Wave#	7
8	0.387	0.241	0.156	0.0020	0.100	1.83	0.024	25.0	0.176	0.106	0.027	7	3.2	4.7	0.073	1.8	0.66	Flat	8
1	0.435	0.241	0.156	0.00225	0.106	2.06	0.0215	25.0	0.171	0.112	0.0235	6	3.4	5.6	0.085	2.3	0.74	Flat	1
9	0.561	0.241	0.156	0.0039	0.140	2.66	0.022	25.0	0.177	0.149	0.025	7	5.6	12	0.163	1.7	0.95	Flat	9

* Data for Run H-3 apply only to the flow over the flat section of the sand wave, which covered major part of flume.

\# Data for Runs H-7, 6, and 7 are composites for flat and dune-covered sections together.

TABLE 8

SUMMARY OF EXPERIMENTS BY VANONI AND BROOKS IN 33.5-INCH FLUME, 1956

Sand No. 4, D_g = 0.137 mm, σ_g = 1.38

Run No.	Q Dis-charge cfs	d Depth ft	r Hydr. Radius ft	S Slope	U_* Shear Vel. ft/sec	U Ave. Vel. ft/sec	f Frict. Factor	T Water Temp. °C	r_b Bed Hydr. Radius ft	U_{*b} Bed Shear Vel. ft/sec	f_b Bed Frict. Factor	No. of Sed. Disch. Samples	$\bar{C}$ Sed. Disch. Conc. gr/l	G Sed. Dis-charge lb/min	Analysis of Sediment Load D_g mm	Analysis of Sediment Load σ_g	F Froude No.	Bed Condition	Run No.
Depth Range: 0.203-0.302 ft																			
2-9	0.510	0.238	0.203	0.00141	0.096	0.77	0.124	23.4	0.230	0.102	0.140	20	0.037	0.071	0.086	1.44	0.28	Dunes	2-9
2-3	0.615	0.243	0.207	0.00204	0.117	0.90	0.133	24.5	0.238	0.125	0.153	13	0.24	0.54	0.094	1.53	0.32	Dunes	2-3
2-8	0.715	0.240	0.205	0.00280	0.136	1.07	0.129	25.2	0.233	0.145	0.147	16	1.15	3.1	0.096	1.49	0.38	Dunes	2-8
2-1	0.855	0.240	0.205	0.00278	0.135	1.28	0.090	25.5	0.231	0.144	0.101	12	1.9	6.0	0.086	1.56	0.46	Dunes	2-1
2-7	0.930	0.237	0.203	0.00277	0.134	1.40	0.074	22.4	0.227	0.142	0.083	20	2.2	7.6	0.086	1.54	0.51	Dunes	2-7
2-6	1.00	0.249	0.211	0.00246	0.129	1.44	0.064	27.4	0.236	0.137	0.072	8	1.4	5.3	0.086	1.64	0.51	Dunes (s.w)	2-6
2-17D	1.17	0.302	0.248	0.00201	0.127	1.39	0.067	18.9	0.284	0.136	0.077	4	2.2	9.8	0.092	1.49	0.44	Dunes (s.w)	2-17D
2-17F	1.17	0.203	0.177	0.00276	0.125	2.07	0.029	18.9	0.186	0.133	0.031	8	3.0	13	0.097	1.46	0.81	Flat (s.w)	2-17F
2-2	1.38	0.233	0.200	0.00205	0.115	2.13	0.0235	23.5	0.208	0.117	0.024	22	2.5	13	0.095	1.60	0.78	Flat (s.w)	2-2
Depth Range: 0.524-0.553 ft																			
2-12	1.21	0.541	0.390	0.00039	0.070	0.80	0.061	24.6	0.488	0.078	0.076	16	0.0033	0.015	0.097	1.66	0.19	Dunes	2-12
2-5	1.54	0.528	0.383	0.00070	0.092	1.04	0.063	23.4	0.480	0.104	0.079	16	0.068	0.39	0.085	1.60	0.25	Dunes	2-5
2-10	1.87	0.549	0.394	0.00105	0.116	1.22	0.072	21.9	0.505	0.131	0.092	16	0.21	1.5	0.084	1.49	0.29	Dunes	2-10
2-11	2.23	0.536	0.387	0.00122	0.123	1.49	0.055	25.2	0.485	0.138	0.069	16	0.67	5.6	0.078	1.52	0.36	Dunes	2-11
2-13D	2.65	0.553	0.396	0.00102	0.119	1.72	0.038	20.7	0.482	0.126	0.046	4	1.45	14	0.088	1.50	0.41	Dunes (s.w)	2-13D
2-16F	3.50	0.524	0.381	0.00107	0.115	2.39	0.0185	16.5	0.408	0.119	0.0195	-	-	-	-	-	0.59	Flat (s.w)	2-16F
2-4	3.84	0.544	0.391	0.00107	0.116	2.53	0.0170	24.9	0.416	0.120	0.0180	16	1.15	16.5	0.124	1.45	0.60	Flat	2-4

D Dune section in runs with a long sand wave.
F Flat section in runs with a long sand wave.
s.w. Sand wave(s) in system.

TABLE 9

Summary of Sand Size Distributions

Sand No.	D_g Geom. Mean mm	σ_g Geom. Std. Deviation	D_{35} (35% finer) mm	D_{65} (65% finer) mm	D_s Mean Sed. Diameter mm
1	0.145	1.11	0.140	0.151	0.16
2	0.088	1.17	0.084	0.094	0.16
3	0.145	1.30	0.130	0.159	-
4	0.137	1.38	0.123	0.155	-
5	0.152	1.76	0.123	0.191	-

Note: $D_g = \sqrt{D_{84} D_{16}}$; $\sigma_g = \sqrt{\frac{D_{84}}{D_{16}}}$

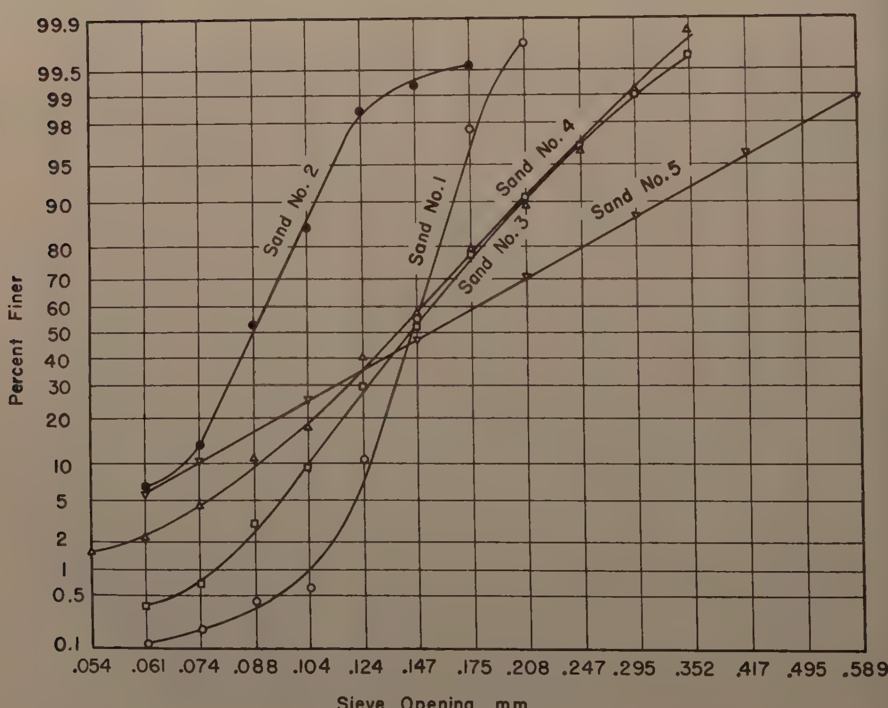


Fig. 26. Sieve analyses of all the sands used. (See also Table 9 above)

spread in the grain sizes is so large that a mean sedimentation diameter is meaningless.

Sands 3 and 4 were obtained from a local foundry supply company and were not further processed except for washing. Sand No. 5 was a synthetic preparation designed to give a logarithmically normal size distribution with $D_g = 0.15$ mm and $\sigma_g = 1.8$. Under microscopic analysis sand No. 4 was found to contain more than 99% quartz grains of subrounded shape. The other sands are believed to be similar in composition.

Since the apparatus and procedure used in the recent experiments were practically identical with that previously used, only the changes will be noted herein. The experiments listed in Table 7 were performed by Dr. George Nomicos in the 10.5-inch flume. Prior to his work a new venturi meter with a 3-inch throat was installed just upstream of the transparent tube (see Fig. 1). The total-load sampler was also improved by using a smaller tube at the bottom of the loop to prevent any possibility of sand storage in the sampler.

The procedure followed by Dr. Nomicos in making his experiments was practically identical to that followed by the writer. At least 300 lbs of sand was used, or enough to cover the bed to a depth of about one inch, which was twice as deep as in the writer's experiments. In presenting his results, in those cases where a long flat sand wave appeared, he did not report separate data for both the rough and the smooth section, but gave only composite averages for the entire flume (except for Run H-3, as noted in Table 7).

The runs listed in Table 8 were performed in a flume 33.5 in. wide and 60 ft long. Hydraulically it is the flume that Vanoni (4) used in his early experiments on transportation of suspended sediment, but structurally it has been recently rebuilt to facilitate the adjustment of slope. Schematically it is now like the 10.5-inch flume. This structural modification has greatly facilitated making experiments of the type presented herein because the slope can be easily adjusted even with the water flowing. These experiments were performed and analyzed by the writer with guidance from Dr. Vanoni, and assistance from Mr. Hugh S. Bell, Jr.

The procedure used in the larger flume is again practically the same as that already reported on. One important difference is that there is no temperature control. Even though the variation of temperature during each run was still slight owing to the large volume of water in the system, the run temperatures range from 16.5 to 27.4^{0}C. The flume was charged with 2500 lbs of sand No. 4, which was enough to cover the entire bed to a depth of 0.15 ft or nearly 2 in. With this depth of sand there was no difficulty with bare spots on the bed except in very extraordinary cases, such as in front of a large sand bar or sand wave. As with the 10.5-inch flume, the sediment samples were withdrawn from the vertical section of pipe a few feet above the pump. Three different samplers were used having 2, 3, or 4 sampling tips connected to a manifold, the choice of samplers used depending on the total rate of flow.

The arrangement of the data in Tables 7 and 8 is identical to that of Table 1 except that the column entitled "Water Surface Condition" has been replaced by two columns giving the geometric mean sieve diameter (D_g) and geometric standard deviation (σ_g) for the composite of the sediment discharge samples for each run. As before, the size distributions were determined by dry sieve analysis. D_g is practically identical to the median diameter (D_{50}) because the sand sizes are very nearly logarithmically normally distributed. In cases where there is a slight difference between the two quantities, D_g has been

determined by the method suggested by Otto.[3]

The terminology used in the tables to describe the bed condition is the same as before except the word "flat" is used in place of "smooth," in line with Dr. Barton's suggestion. The term "sand wave" is used for the condition described on the bottom of page 668-7 where the sand spreads itself nonuniformly in the flume, creating a thick flat section in one part and a thinner dune section in the other. Dr. Barton has used the term "sand bar" to describe the same phenomenon observed in his flume studies. A typical front of a sand wave is shown for Run 2-17 in Fig. 27. In the direction of flow this is the point of transition from flat bed (small depth) to dune-covered bed (larger depth); the transition from dunes to flat bed is always gradual. As Dr. Barton points out, the sand front was always observed to be diagonal to the flow for some unknown reason. For some of the runs in the large flume, several secondary sand fronts or sand bars were observed in addition to the main one, usually following behind it.

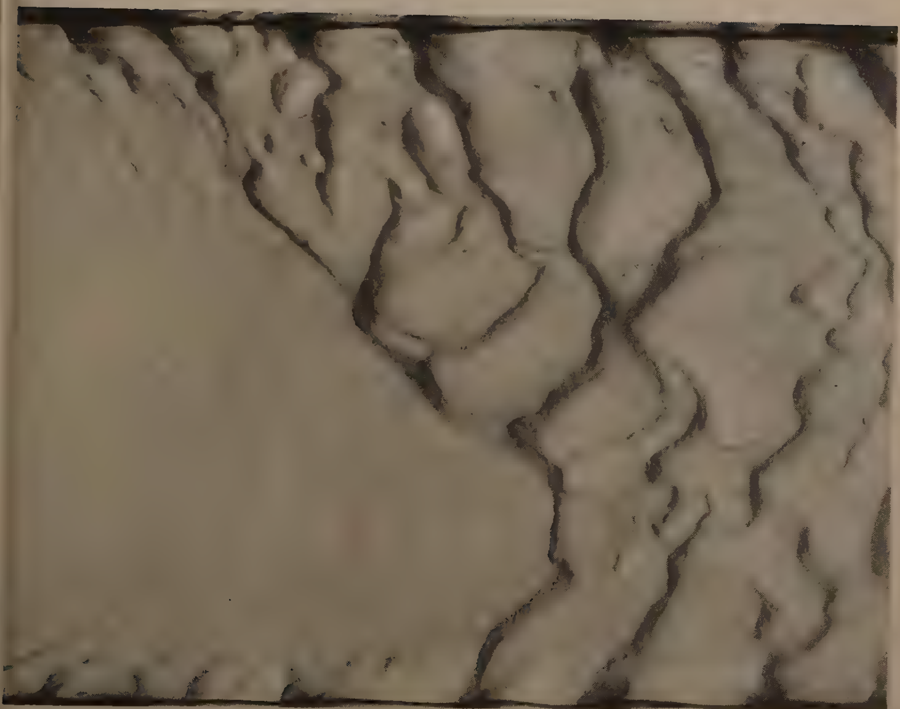
The data in Tables 7 and 8 are presented graphically in Figs. 28 through 32 in a slightly different manner from the original paper. All the experiments made by Dr. Nomicos were at the constant depth of 0.24 ft. and all the runs made by Dr. Vanoni and the writer were intended to have depths of 0.24 or 0.54 ft. With the depth thus limited to two selected values, more attention could be focused on exactly what changes take place as the velocity increases with depth constant. In effect, a horizontal cross-section is being taken through Fig. 8.

First of all, Fig. 28 shows how the slope, S, the shear velocity for the bed U_{*b}, and the bed friction factor f_b changed with velocity for a nominal depth of 0.24 ft for runs made with Sands 1, 3, and 4 in either of the two flumes. Notice that Runs 3, 8, 9, 10 of the original data are also included for comparison with the new data. It is noteworthy that the points from three different investigations in the two different-sized flumes on three sands with different geometric standard deviations may easily be fitted with common curves for S and f_b.

Similar graphs for d = 0.54 ft (Sand No. 4 in the 33.5-inch flume), and d = 0.24 ft (Sand No. 5 in the 10.5-inch flume), are shown in Figs. 29 and 30 respectively. In Fig. 31 are presented the concentrations measured for the runs in each of the three categories above. Points for some of the new runs (2-17D, 2-17F, and H-3) are not included in Figs. 28-31 because the depths lie outside the range used for the figures.

The graph in Fig. 32 is a composite of all of the data for Sands 1, 3, 4, and 5 and is similar to Fig. 10. The water discharge Q and the sediment discharge G have been replaced by q and q_s, the same quantities respectively per unit width. As before, the logarithmic cycles in the abscissa are twice as long as those in the ordinate. Labeled next to each point is the depth. The friction factors, which are also labeled adjacent to the points in Fig. 10, have been omitted because they do not appear to define a simple pattern of contours. On the other hand, contour lines for constant depths of 0.24 and 0.54 ft have been drawn in Fig. 32, and they show the same general tendency as the constant depth lines sketched previously in Fig. 12. Because of the different sands used for a depth of 0.24 ft, it was necessary to plot three slightly

3. "A Modified Logarithmic Probability Graph for Interpretation of Mechanical Analysis of Sediment" by George H. Otto, Jour. of Sed. Petrology, Vol. 9, No. 2 (1939) pp. 62-76.



Flow direction ⟶

Flat Section		Dune Section
0.203	Depth d, ft	0.302
2.07	Mean velocity, U, fps	1.39
0.031	Bed friction factor, f_b	0.077
0.15	Ave. thickness of sand bed, ft.	0.10

Fig. 27. Typical front of a sand wave (Run 2-17). Photograph shows full width of 33.5-inch flume.

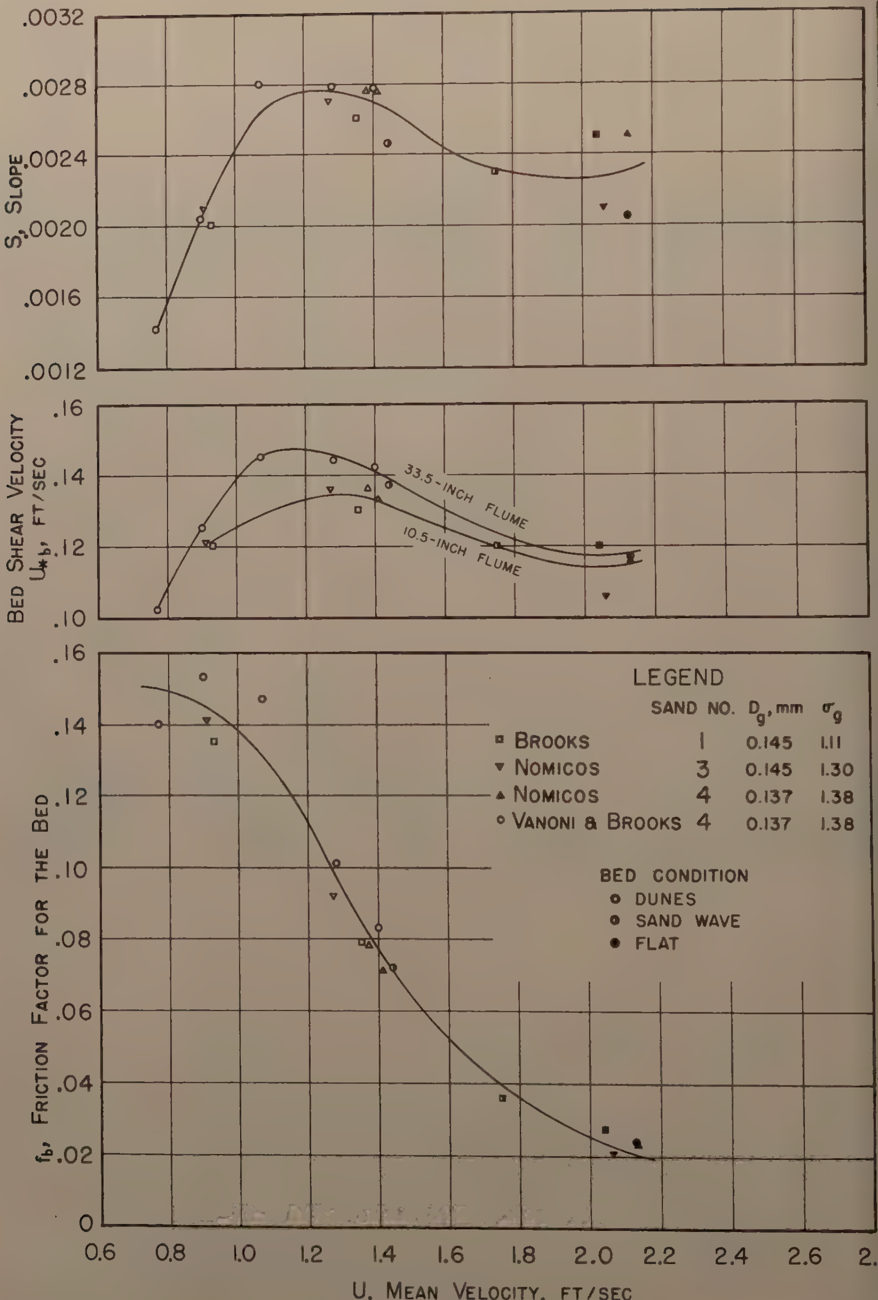


Fig. 28. Variation of S, U_{*b}, and f_b with U for depth = 0.233 to 0.250 ft for Sands No.1, 3, and 4 in 10.5-inch flume (Brooks, Nomicos) and Sand No. 4 in 33.5-inch flume (Vanoni and Brooks).

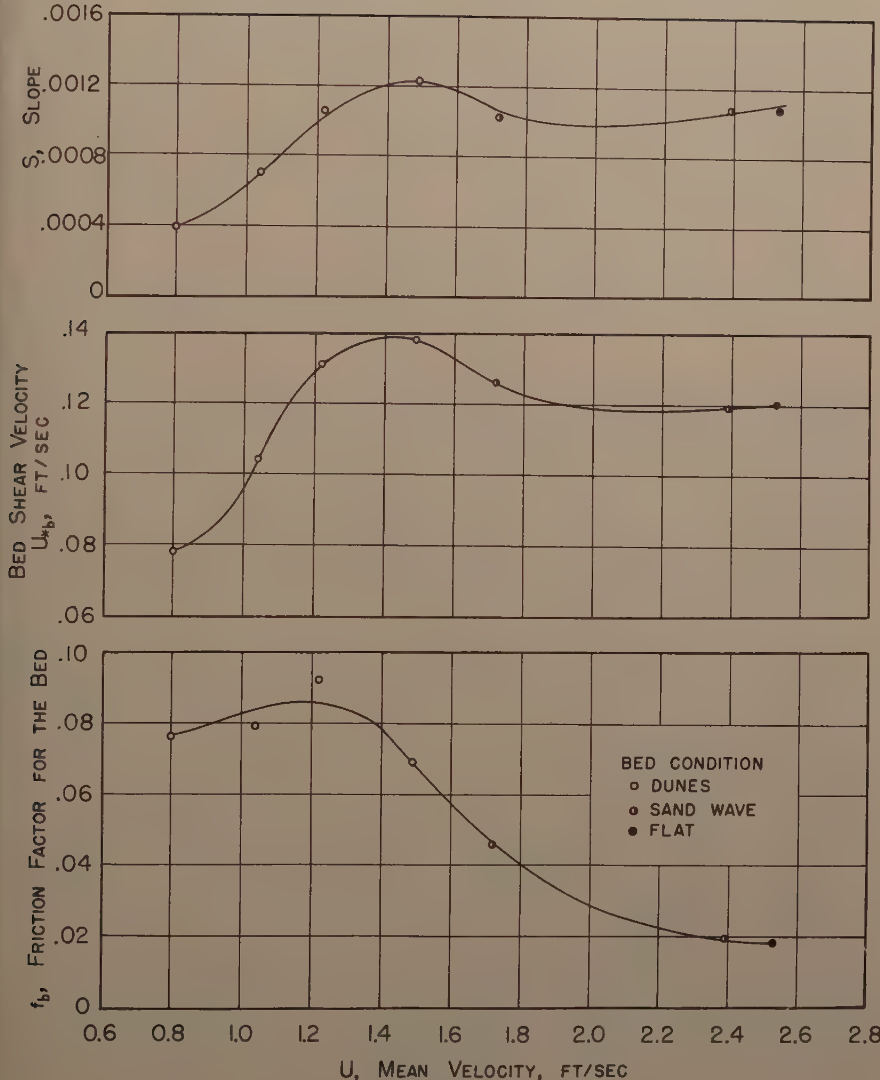


Fig. 29. Variation of S, U_{*b}, and f_b with U for depth = 0.524 to 0.553 ft for Sand No. 4 in 33.5-inch flume.

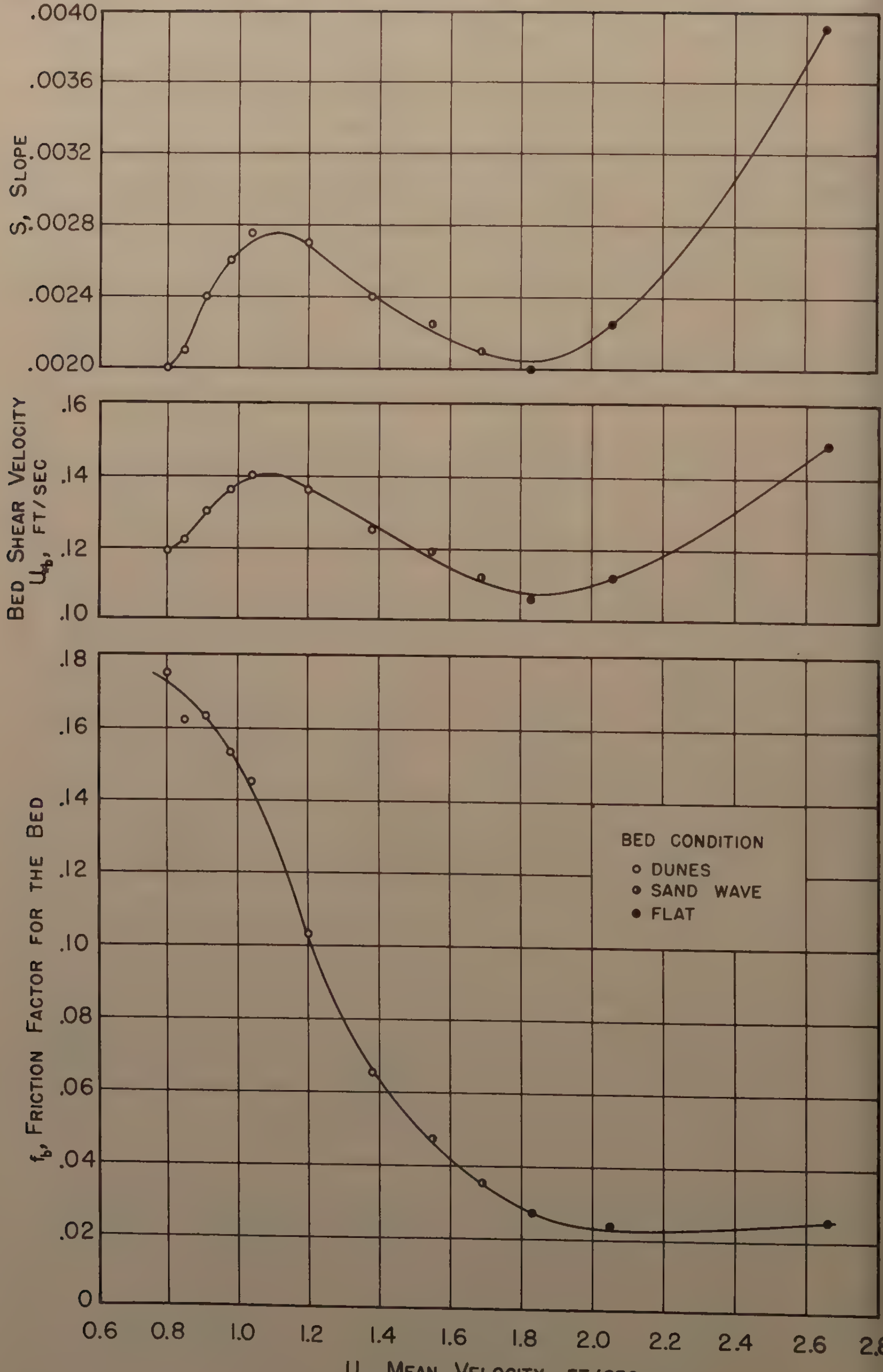


Fig. 30. Variation of S, U_{*b}, and f_b with U for depth = 0.241 ft for Sand No. 5 in 10. 5-inch fl...

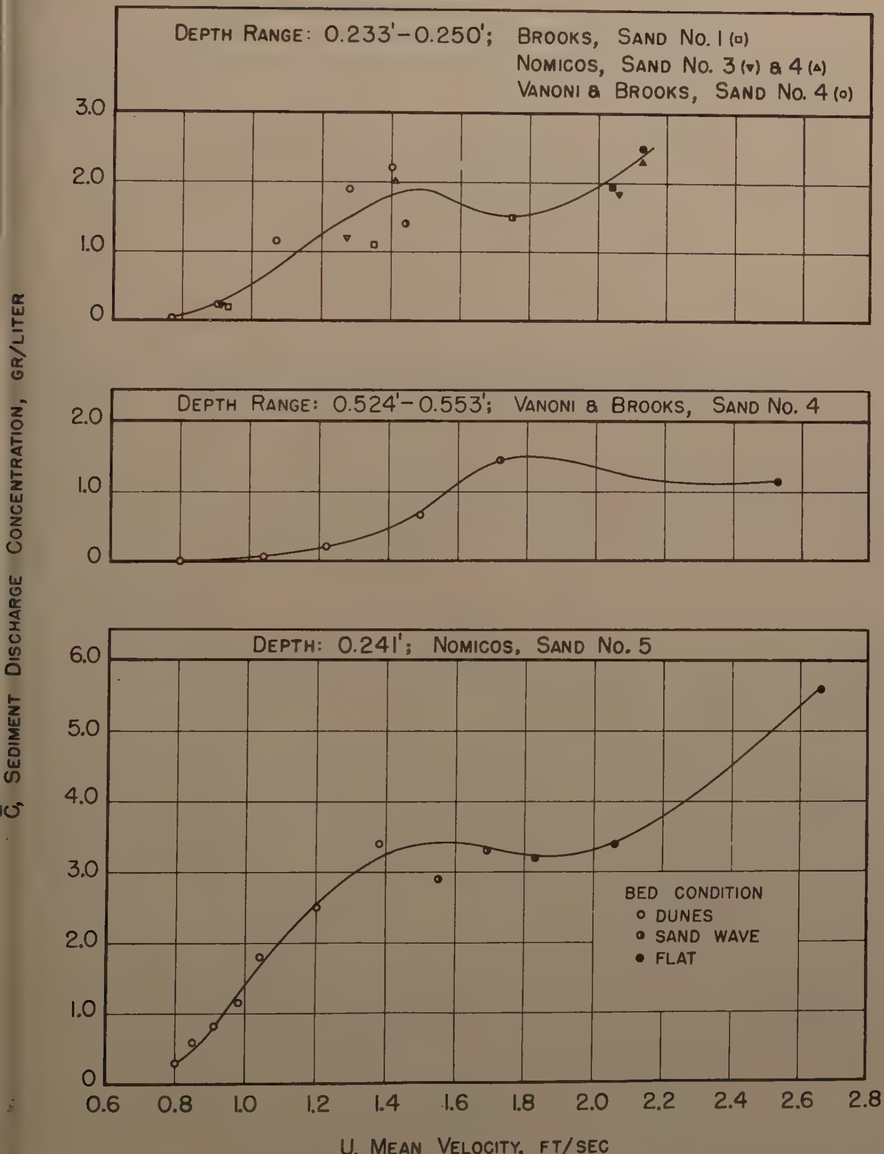


Fig. 31. Variation of $\overline{C}$ with U for various sands at constant depths as indicated.

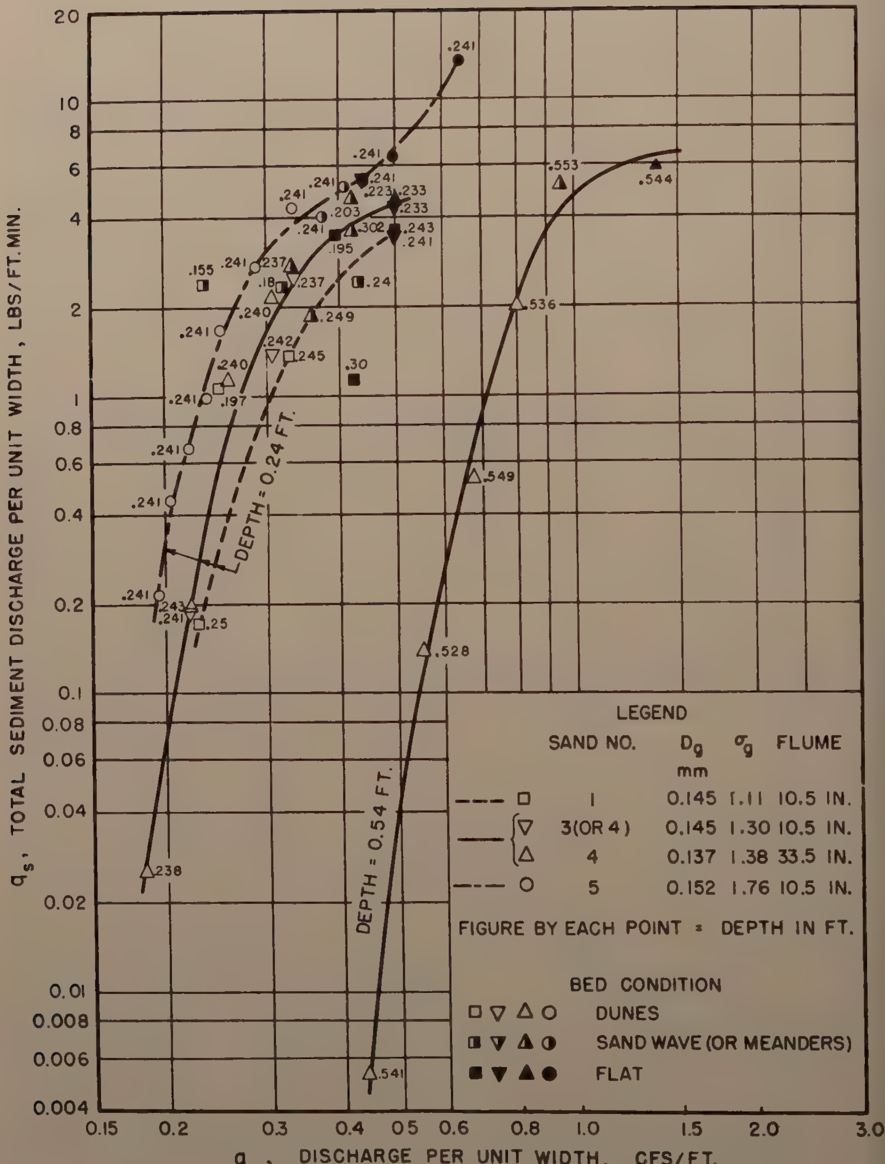


Fig. 32. Relation of depth to q and q_s for various sands.

different contours for this depth. In these experiments, the unit sediment discharge q_s increases somewhat when σ_g increases, with Q, d, and D_g staying nearly constant. Unfortunately, the graph is not free from temperature effects, as the range covered is 16.5°-27.5°C. It is expected that the scatter would be less if the temperature were constant, as in Nomicos' Series II (Sand No. 5) where T = 25.0-26.0°C.

For more complete and detailed information and analysis of the work carried on under the contract with the Corps of Engineers, Omaha Division, and under the grant from the National Science Foundation, the reader is referred to the final research reports.[4,5]

Discussion of Results

Verification of Previous Conclusions. For the sake of brevity and to avoid duplication with the original paper, the writer will not discuss the new results in detail, hoping that the reader will determine for himself how well the new data support the previously stated conclusions. Suffice it to say here, however, that the above data support all of the previous conclusions of the writer with the exceptions as follows:

a) In conclusion 4(b), the writer previously stated, "For a given slope and discharge it was found that two depths of flow were possible." Because of the way the runs were scheduled in the recent experiments, no new evidence on this conclusion was obtained. However, good evidence presented by Dr. Barton (page 841-10) indicates that this conclusion may not be generally true. It is hoped in the future to schedule some runs which will definitely provide further evidence on this point.

b) It was stated as conclusion 4(f) that "When d is increased holding U constant, $\overline{C}$ decreases slightly, and f_b does not change appreciably in the range of conditions covered." With a wider range of depth now obtained, it was found that f_b is substantially lower for the larger depth for a given velocity in those cases where the bed was covered with dunes. (Compare Figs. 28 and 29.)

Conclusions 4(g) and 4(h) of the original paper compare the results for two different sand sizes, and therefore cannot be checked against the new data presented here.

Discussion of Conclusion (1). Without exception, the discussers challenged the writer's basic conclusion. Conclusion (1) states (in part): "For the laboratory flume it was found that neither the velocity nor the sediment discharge concentration can be expressed as a single-valued function of the bed shear stress, or any combination of depth and slope, or bed hydraulic radius and slope. . . ." Dr. Lin is certainly correct when he states, "This statement should be read with the understanding that, among other factors, given temperature, channel, and sediment are implied. . . ." Among the discussers, Messrs. Carlson and Mostafa, Liu, Lin, and Blench doubt the validity of the

4. Final Report on Sediment Transportation Studies to Corps of Engineers, Omaha Division, by V. A. Vanoni and N. H. Brooks, Hydrodynamics Laboratory, California Institute of Technology, 1957.
5. Final Report on Studies of Subaqueous Dunes to National Science Foundation, by V. A. Vanoni and N. H. Brooks, Hydrodynamics Laboratory, California Institute of Technology, 1957.

experiments themselves. Messrs. Einstein and Chien, and Barton appear to accept the findings of the flume experiments, but doubt the generality of the findings, feeling perhaps that some peculiar condition resulted in the findings in the flume experiments.

The various criticisms advanced regarding apparatus and procedure have already been answered in a previous section. To the criticism of lack of sufficient number of runs to substantiate the original conclusion, the writer's reply lies in the additional data presented above. For example, Dr. Lin has pointed out that the writer did not actually have two runs with exactly the same depth and slope, but having different velocities, at the same temperature as well. However, Dr. Nomicos has performed a pair of runs (Series I, Runs A and C) which have exactly the same depth, slope, and temperature; yet the velocity in Run A is 2.06 ft/sec while in Run C it is 0.91 ft/sec. The validity of conclusion 1, at least for the laboratory flumes, is certainly conclusively demonstrated by Figs. 28, 29, and 30. The curves representing slope as a function of velocity for a constant depth have very definite inflections in them for d = 0.24 ft and a long horizontal section for d = 0.54 ft. Therefore, one cannot expect to choose a value of S and find only one corresponding value of velocity from the graphs. Similarly, it is apparent that U cannot be uniquely determined from the bed shear velocity, $U_{*b} = \sqrt{\tau_{ob}/\rho}$. The cause of the surprising results, as indicated in Conclusion 2, is clearly the extreme variations of the bed friction factor f_b.

Reply to Liu's Discussion. The writer concurs with Dr. Liu in his analysis of the resistance and transport problem starting with his Eq. (b) (on page 841-31) leading up through the paragraph which contains Eq. (f), understanding that Eq. (f) applies only to the example of rigid boundary open-channel flow with no sediment. However, in the next paragraph the writer believes that Dr. Liu has reasoned incorrectly about the uniqueness of the functional relationship between the various variables. The writer must vigorously disagree with the discusser, even on logical grounds, when he makes the statement, "Such a statement (referring to the writer's Conclusion 1) is in contradiction to his other statement 'f_b and $\overline{C}$ are uniquely determined by U and d' which means that the slope is also uniquely determined by U and d by using Eq. (e)." The same kind of misinterpretation of the writer's meaning of uniqueness and independent variables leads to other implausible statements in the first and fourth paragraphs following the paragraph just referred to.

Near the end of his discussion (12 lines above Eq. (g)), Dr. Liu challenges another statement of the writer. It should be noted that the quotation given in the discussion is not a correct quotation from the original paper (second paragraph under "Experimental Results"), and that the misquoted statement has an altered meaning as it has been taken out of context.

Dr. Liu disagrees also with conclusion 4(d) which states that, "For a given Q, the largest bed friction factors are associated with the lowest values of G." The discusser believes that the dunes will not develop at the lowest transportation rates although he has found experimentally[6,7] that ripples start forming on a smooth bed practically as soon as the sediment starts to move. It has been the writer's experience also that if there is any

6. "The Present Status of Research on Sediment Transport" by Ning Chien, Trans. ASCE, Vol. 122 (1956) p. 833.
7. Ibid, Discussion by H. K. Liu.

transportation at all, large dunes will grow in fine sand if sufficient time is allowed. For this reason conclusion 4(d) should not be unreasonable. However, from Fig. 29 it may be noted that for the runs with d = 0.54 ft the friction factor f_b at the lowest velocity is slightly less than the maximum (vide Conclusion 4(e)). For Run 12 (d = 0.541 ft, V = 0.80 fps) the friction factor f_b was 0.076, which is still a very large value considering that G is only 0.015 lbs/min (or $\overline{C}$ = 3.3 ppm). For this run the flume was run continuously for 54 hours; starting from a flat bed, it took about 24 hours to reach equilibrium, and during the remaining 30 hours the equilibrium appeared very stable.

The writer welcomes Dr. Liu's brief analysis of dune velocity data. His Fig. a confirms the writer's observation (5) that the velocity of advance of the dunes actually decreases as the grain size gets smaller and presumably the sediment load gets larger.

Reply to Barton's Discussion. It is encouraging to know that Dr. Barton has been making experiments at Colorado A. & M. College which are similar to the ones being made by the writer, and that his results are in many ways like those reported herein. The data that he has presented in his Fig. 1 (page 841-11) certainly demonstrate the same general behavior of the friction factor that has been noted by the writer. In comparing data for different depths of flow it was found that the region which Barton has labeled "sand bars on beds" on Fig. 1 moves to the left as the depth decreases. It has been puzzling to the writer to note that even in comparing the 10.5- and 33.5-inch flumes with the same depth of flow that the interval of velocity giving sand bars or sand waves is not exactly the same in the two flumes, being a little bit higher for the wider flume.

Barton's Fig. 2 perhaps implies that the relationship between depth, slope, and discharge is simpler than it really is, even for his flume. If all the data published by Barton and Lin[8] are plotted on a graph like Fig. 2 (p. 841-12) the lines of constant slope cannot be drawn as straight lines like lines (A) and (B) of that figure. In particular along line (B) for S = .0015 to .0017 the slope actually drops to the order of .0012 or less in the gap between discharges of 3 and 6 cfs in Fig. 2. This is indicated by the following runs:[8]

Run No.	Q cfs	d ft.	S	T °C	Bed
29	5.8	0.60	0.00121	25	Flat
31	4.2	0.41	0.00123	26	Flat

Run 29 falls on line (B), while Run 31 falls below; on the other hand one is led to expect from Barton's Fig. 2 that these points would fall between curves (A) and (B) on the basis of their observed slopes.

In spite of this deviation, the writer concurs with Dr. Barton that the Colorado A. & M. data do not support conclusion 4(b). However, some of their results indicate that for a given depth and slope more than one equilibrium discharge could probably be found in the flume at Colorado A. & M. (cf. writer's Conclusion 1). Consider the following runs:[8]

8. "A Study of the Sediment Transport in Alluvial Channels" by James R. Barton and Pin-Nam Lin, Report No. 55 JRB2, Civil Engineering Department, Colorado A. & M. College, March, 1955.

Run No.	Q cfs	U fps	d ft	S	U_{*b}[9] fps	f_b[9]	T °C	Bed
31	4.2	2.56	0.41	0.00123	0.118	0.0169	26	Flat
27	2.1	1.31	0.40	0.00140	0.131	0.080	24	Dunes
16	1.9	1.19	0.40	0.00158	0.140	0.110	22	Dunes

When the depth is constant and the velocity decreases, the slope actually increases because the friction factor for the bed increases so rapidly. As illustrated in Fig. 28, this must be a temporary trend in S; if the velocity were further reduced below 1.19 fps (Barton Run 16), the slope must sooner or later drop also, thus repeating some slopes already obtained for higher velocities. Therefore, although no two of their runs have exactly the same depth and slope with two different discharges, it is seen from the data above that such a result would undoubtedly have occurred had a complete sequence of runs been made at a depth of 0.40 ft in the flume at Colorado A. & M.

Reply to Discussion by Carlson and Mostafa. The analysis, presented by Messrs. Carlson and Mostafa, appears to be grossly oversimplified. In their Fig. 1 the slope is plotted against concentration (expressed as $G/60\,\gamma Q$). The discussers have given no logical reason why the slope should be so simply related to the concentration, and the scatter of the points indicates oversimplification. Fig. 2 in the same discussion shows how the observations for dunes and those for flat beds plot along two different lines in a graph of $r_b S/D$ versus concentration. A graph such as this does not solve a problem of whether there will be dunes or not in a given case; the engineer has no basis for choosing which curve to read the answer on. The further inadequacy of Fig. 2 is illustrated by the discussers' conclusion: "It follows that for the same hydraulic conditions as given by r_b, S and Q, two different ratios of sediment to water discharge might have been produced, one with a smooth bed, and the other time with a dune formation." If the analysis leading to the graph in Fig. 2 were sound, this would be a logical conclusion. However, the fact that this conclusion is completely contrary to the observed results does not cast doubts upon the writer's conclusions but on the analysis instead. When values of r_b, S, and Q are selected, the friction factor is automatically determined; it follows then that it is impossible to produce either a smooth bed or a dune-covered bed when the friction factor is restricted to one value.

The writer does not believe that the shortcomings of the graphs of Messrs. Carlson and Mostafa can be remedied by information on the grain sizes of the sediment load. Nonetheless, since this was an omission of the original paper, the available information on the size distributions of the sediment samples is given in Table 10. Unfortunately, the data are incomplete, but the extreme range of transportation rates is covered so that the missing values could be expected to lie within the range of the values reported therein. The observed variations in D_g and σ_g between runs is believed to be of little consequence.

Reply to Lin's Discussion. Another dimensional analysis, presented by Dr. Lin, is based largely on a report[8] which was issued after the writer's original paper had been submitted for publication. Since Dr. Lin presents only the main results of his theoretical analysis and refers also to a paper being

9. Computed by writer; other data from Ref. 8.

TABLE 10

Analysis of Sediment Load for Runs in Table 1.

Run No.	$\bar{C}$ gr/l	D_g mm	σ_g
Sand No. 1			
3	1.95	0.147	1.10
4	2.45	0.141	1.13
6	2.45	0.145	1.11
7	2.15	0.149	1.09
10	0.2	0.146	1.10
12	0.72	0.128	1.16
Sand No. 2			
21a	4.9	0.079	1.22
24	4	0.070	1.28
26	0.19	0.073	1.36
30	1.75	0.069	1.30

prepared by himself and Dr. Barton, a detailed discussion of their analysis would not be appropriate here; hence, the following comments are confined to material presented in his discussion.

Dr. Lin's interpretation of the writer's data depends on his Fig. L-1. It should also be noted that through some error the point for Run 26 at the far left of Fig. L-1 which is plotted at about $\frac{w\bar{C}}{UF^2} = 0.01 \times 10^{-3}$ should be plotted at about 0.1×10^{-3}. It appears that the drawing of the entire family of curves has been strongly influenced by this erroneous point; the writer does not see how the curves could be replotted or the discussion of Dr. Lin's revised to accommodate this error. Furthermore, the discusser (on page 841-36) implies that existing laboratory and field data generally fall in the range $\frac{w\bar{C}}{UF^2} < 0.07 \times 10^{-3}$, and thus exhibit a different behavior from the writer's data which fall entirely above this number. However, by cursory examination, and a few calculations on the data presented by Drs. Barton and Linn,[8] this was not found to be the case, as values as high as 0.4×10^{-3} were calculated. In regard to Dr. Lin's comments about the von Karman's constant, values measured for some of the experimental runs are presented in the writer's thesis.[5] Further discussion of the variability of the von Karman constant and its interrelation with the suspended sediment load has been presented by Nomicos.[10]

Two of the discussions (Einstein and Chien, and Lin) refer to the work of Meyer-Peter and Müller.[11] The latter have divided the energy gradient into

10. "Effects of Sediment Load on the Velocity Field and Friction Factor of Turbulent Flow in an Open Channel" by George N. Nomicos, Ph.D. Thesis, California Institute of Technology, May, 1956.
11. "Formulas for Bed-Load Transport," by E. Meyer-Peter and R. Muller, Internatl. Assn. for Hydr. Structures Res., Proc. 2nd Meeting, Stockholm, 1948, pp. 39-65.

two parts; that due to the grain roughness of the surface and that resulting from the "topography" of the bed. Like Einstein, they have theorized that the rate of bed-load transport is dependent only on friction associated with the grain roughness. Their final formula for the rate of bed-load transportation shows that the transportation rate depends on the shear with a correction factor $\left(\frac{k_s}{k_r}\right)^{3/2}$. They have not suggested how one is to predict what this factor should be, nor have they indicated that there might possibly be any difficulty with multiple solutions. That is, for a given value of shear there may be more than one possible value of this correction factor. Therefore, the writer would like to dispute the statement by Dr. Lin to the effect that Meyer-Peter and Müller have published a finding similar to the writer's conclusion 1.

Reply to Blench's Discussion. The writer wishes to thank Professor Blench for pointing out the misunderstandings in regard to regime theory. As seen from reference 15, the regime theory certainly does include the shear; but, on the other hand, from the form of the equations comprising generalized regime theory as reported by Professor Blench, there is no possibility for nonuniqueness of the type observed by the writer in the laboratory (Conclusion 1). The writer certainly concurs with Professor Blench, though, when he implies that the discharge and the grain size and other characteristics of the alluvium are to be considered independent variables, and the depth, width, and slope of a stream are considered dependent variables (vide (15), p. 395).

In regard to the last misunderstanding enumerated by Prof. Blench it should be pointed out that the "two present theories" referred to under Conclusion 1, were those of Lane and Kalinske (13), and Einstein and Barbarossa (14). It was not the intention of the writer to dispute regime theory, which does not deal explicitly with sediment load.

Reply to Discussion by Einstein and Chien. Drs. Einstein and Chien have presented a very thorough and thought-provoking analysis of the writer's paper. Their analysis is based primarily on an interpretation of the experimental results on the basis of Einstein and Barbarossa's theory for river channel roughness (14). The discussers demonstrate how the bar-resistance curve of Einstein and Barbarossa can actually lead to multiple solutions for certain values of the depth, slope, and grain size. On page 841-15 they state, "The existence of such uncertainty in the prediction of discharge and sediment load in very limited ranges of conditions was known previously, but not much significance was attached to the fact because of the rather rare occurrences of these cases." Inasmuch as the discussers did not offer any reference in connection with this statement, it may be inferred that it was previously known by only a few people, and that whether or not these are rare occurrences remains to be demonstrated. If such cases are not rare, then the usefulness of the proposed bar-resistance curve will be limited by the fact that one will not know which of two or more possible solutions to choose as the correct answer.

The writer thinks that the uncertainty of solutions derived from the bar-resistance curve may be more widespread than indicated by the curve itself because of the large range of scatter of the original river data on which the bar-resistance graph is based (Fig. 3 of Ref. 14). The tremendous effects of this scatter on a typical computed rating curve have already been ably demonstrated with numerical examples by Doland and Chow, and Inglis in discussions of Einstein and Barbarossa's paper (Ref. 14). On Fig. 33 the

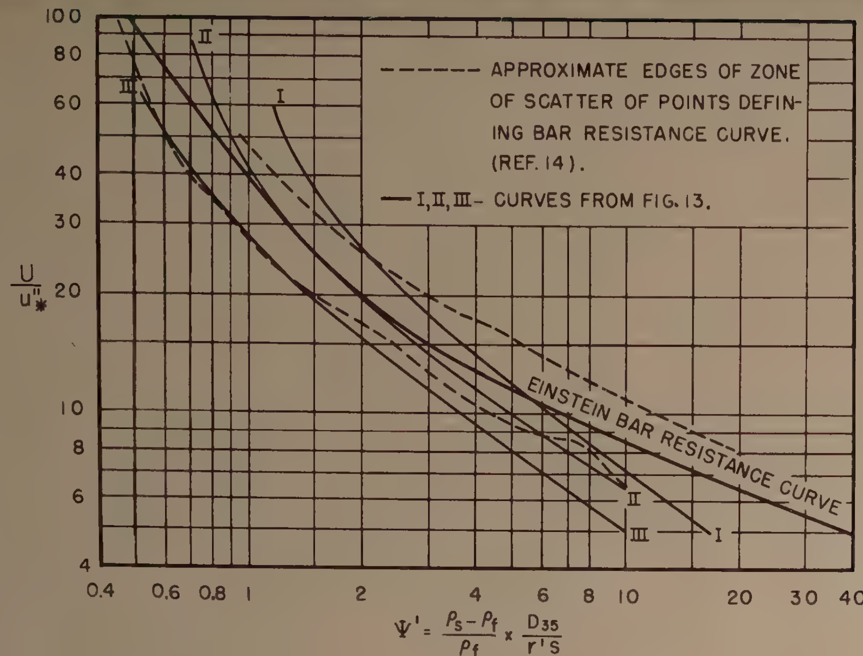


Fig. 33. Zone of scatter of points defining the bar-resistance curve, showing the possible difficulty in defining solutions for typical resistance problems represented by the intersections of curves I, II, and III with the bar-resistance curve.

bar-resistance curve has been plotted as a solid line with two dashed lines on each side indicating the approximate range of scatter as shown in the original derivation of the curve. In addition, Fig. 33 shows curves I, II, and III from Fig. 13 of the discussion by Drs. Einstein and Chien. Allowing that possible solutions may be anywhere within this band of scatter, and not necessarily precisely on the bar-resistance curve, one finds a wide range of possible solutions for the curves I and II shown. For example, curve II lies within the zone of scatter all the way from $U/u'' = 9$ to 50. Therefore, the writer does not believe that the discussers can argue that the occurrences of such uncertainty are rare merely on the basis of the bar-resistance curve.

The discussers have noted that the flume values of U/u''_{*b} and ψ', do not follow the bar-resistance curve for natural rivers (see Fig. 17). They have suggested that the discrepancy is due to the unnatural uniformity of the material used in the flume. Following this suggestion, experiments were made with sands of varying geometric standard deviation (Fig. 26 and Table 9) with the results as given in Tables 7 and 8.[12]

Calculations of ψ' and U/u''_{*b} were made for all the runs reported in Tables 7 and 8 as well as for all the data presented in Table 1 of the original paper. In making these calculations the procedure suggested by Einstein and Barbarossa (14) was used, without simplifying assumptions. By a change of parameters and specially derived working graphs the writer was able to solve their Eq. (7), (8), and (9) without resorting to trial and error. The computed values for the writer's flume data are slightly different from Einstein and Chien's because a distinction was made between D_s, and D_{65} and D_{35}. All of these data are plotted together on Fig. 34. The surprising result is that the sorting of the material makes practically no difference in the bar-resistance relationships for the flume data. Even runs in Series II ($\sigma_g = 1.76$) behave almost identically to the writer's Runs 2 through 13 ($\sigma_g = 1.11$). For $\sigma_g = 1.76$, Einstein's sorting coefficient S_0 is 1.46, which is greater than the majority of the values listed for natural rivers in Table 3.

A possible explanation for the apparent inconsistency between flume and river data is the wall effect. Whereas, in the laboratory flumes the maximum width to depth ratio was only about 12, natural rivers commonly have width-depth ratios of the order of 100. In the laboratory, the stream is confined to a straight channel, and the walls are usually smooth. The sidewall correction procedure (6) is supposed to eliminate the wall effect as far as the calculation of the bed friction is concerned, but there is no known correction for the effect of the walls on the sediment transportation and dune configuration.

In the writer's opinion roughness of alluvial channels will have to be expressed basically in terms of a velocity parameter instead of a shear parameter such as ψ used by Einstein and Barbarossa. Because enormous changes in roughness have been observed to occur with almost negligible changes in the bed shear (or U_{*b}), it appears that the latter is a poor choice of independent

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12. The discussers' sorting coefficient S_o is related to the more commonly used geometric standard deviation σ_g in the following way for the logarithmic normal distribution:

$$S_o = \sqrt{\frac{D_{75}}{D_{25}}} = \left[\sqrt{\frac{D_{84}}{D_{16}}}\right]^{0.67} = \sigma_g\ 0.67$$

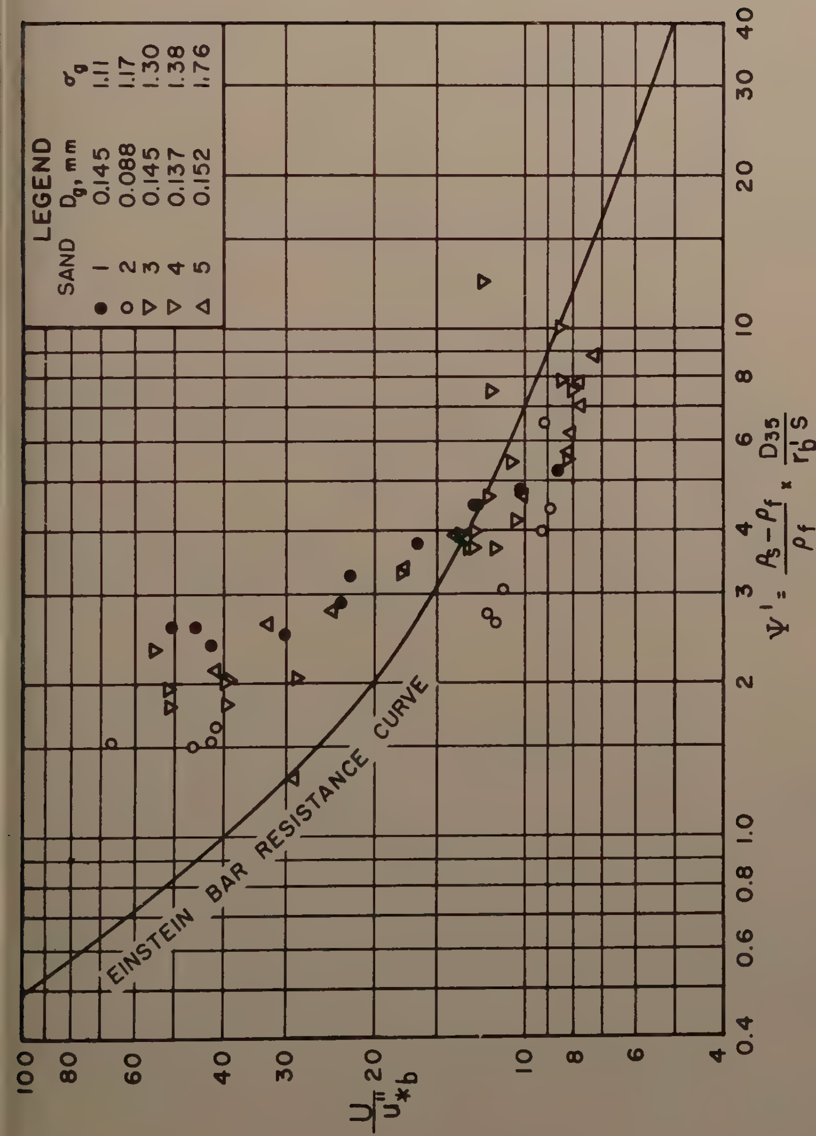

Fig. 34. Comparison of points for flume experiments listed in Tables 1, 7, and 8 with Einstein-Barbarossa bar-resistance curve for natural channels.

variable. In a recent paper, Ali and Albertson[13] have presented an interesting analysis of roughness of alluvial streams based on Reynolds number, which is a velocity parameter.

In the concluding paragraph of the discussion by Drs. Einstein and Chien, the writer's results are compared with Einstein's $\phi_* - \psi_*$ relationship (7). As the derived points fit the Einstein curve reasonably well in most cases, the discussers are led to conclude "that the concept in linking the bed-load transport with the grain resistance of the bed material only is basically sound." In considering this deduction it is well to realize some of the assumptions that have been made in deriving the plotted points from the writer's data presented. In the first place, the functional relationship given in Fig. 19 is basically for bed-load transport. In order to evaluate the bed-load discharge from the writer's measured values of total load discharge, the calculations involve finding the ratio of suspended load discharge to bed-load discharge. The suspended load discharge is computed on the basis of the following principal assumptions:

1. The concentration of suspended material at a distance equal to two grain diameters above the bed (y = 2D) may be considered a boundary condition for the suspended load equation (Eq. 1). This concentration is found by dividing the bed load transport by $(2D)(11.6\, u'_*)$ in which u'_* is the shear velocity associated with the grain resistance only.

2. The velocity distribution is expressed in terms of $\frac{u'_*}{0.40}$, thus implying (a) that the von Karman constant k = 0.40 and, (b) that only the shear due to grain resistance affects the velocity distribution in the vertical.[14]

3. The exponent z of the suspended load equation is given by

$$z = \frac{w}{0.40\, u'_*}$$

where w is the settling velocity and 0.40 is βk (see Eq. (2), p. 668-2). It is implied by this relation that the upward diffusion of the suspended load depends only on u'_*, and not the overall shear velocity U_* (or U_{*b}).

Using these assumptions, the product of concentration and velocity may be integrated from y = 2D to y = d to yield the rate of suspended load transport as a certain multiple of the bed-load transport. Knowing this, the bed load may be computed from the measured total sediment discharge.

Now each one of the three basic assumptions above is open to serious question especially for runs with dune-covered beds. In the first place it is unreasonable to apply the suspended-load equation down to an elevation of two grain diameters when the height of the dunes is of the order of several hundred grain diameters. In the second place, it should be noted that many investigators[6] have shown that the von Karman constant k for sediment-laden streams is substantially less than the assumed value 0.4. Data reported by Barton and Lin[8] also show that assumption 2 is poorly supported in fact for runs with dune-covered beds. In regard to assumption 3 their experiments

13. "Some Aspects of Roughness in Alluvial Channels," by Said M. Ali and Maurice L. Albertson, Dept. of Civil Eng., Colorado A. & M. College August, 1956.
14. Actually u'_* introduced by assumption (1) cancels the same quantity arising from assumption (2) in the final computations in Einstein's method.

further indicate that the theoretical exponent z based even on the total shear velocity U_* is still larger than the observed exponent when the bed is covered with dunes. Since it is not uncommon to have $u'_* < 0.5\ U_*$ the error in the shape of the suspended load distribution resulting from using a value of z which may be more than twice the actual value can be considerable.

Although the gross errors in each of the above mentioned assumptions may tend to cancel out in the interlocking of the assumptions in the computations, the writer still believes that one is not justified in concluding anything very definite about the relation of bed load to suspended load from results of such a tortuous calculation. In fact, the only supporting evidence for Dr. Einstein's analysis of the suspended load in reference 7 is an analysis similar to that described above for experiments made by Dr. Einstein in the same 10.5-inch flume that the writer used at the California Institute of Technology.

Field Observations

Drs. Einstein and Chien have correctly pointed out some of the weaknesses of the field examples cited by the author. There are very few field data which are sufficiently detailed to conclusively verify or disprove conclusions arrived at in a laboratory flume. The main difficulty with the example of the Colorado River at the Grand Canyon is that there is no information on the division of the total suspended load between wash load and bed-material load, and there are no data on slope or grain sizes in the bed. The changes that took place during the spring flood of December, 1940 to June, 1941 probably did not occur in a matter of days, as indicated by Drs. Einstein and Chien, but in the matter of weeks and months. It seems as though this should be sufficient time for scouring the bed and changing the roughness. In this connection, it has been observed in a laboratory flume that a system of dunes can be wiped out in less than one minute by a flow of a higher velocity which is in equilibrium only with a flat bed.

The author recognizes that the scour of the river channel at the Grand Canyon may not be generally true of the entire river system. As pointed out by the discussers, Lane and Borland[15] found that on the Rio Grande the narrow sections of the river tended to scour out during floods whereas the wider sections tended to aggrade. Although it may well be that the overall regime of the river is not changed, still the changes that occur in any given reach must be in response to the changes in the discharge and sediment load entering that reach. Even though the changes may not proceed all the way to a new equilibrium, the alterations of the stream characteristics are certainly in that direction.

The situation at Yuma is certainly complicated by the location of Imperial Dam, 15.8 miles upstream. Not only is there general degradation taking place below Imperial Dam as mentioned by Drs. Einstein and Chien, but there is also local choking of the river channel between Imperial Dam and Laguna Dam (about six miles downstream) due to the sludge returned from the desilting works.[16] By the diversion of essentially clear water from the river

15. Ref. D. of Einstein and Chien Discussion.
16. "River Channel Conditions Imperial Dam to Laguna Dam," Colorado River Front Work and Levee System, U.S. Bureau Reclamation, Region 3, Boulder City, Nevada, Aug. 1954.

to the All-American Canal the remaining water has to carry a higher concentration of sediment below Imperial Dam. In order to move this material down the river and preserve the channel below Imperial Dam it has been necessary to sluice the channel for several hours every week or two with a relatively high discharge (20,000 cfs or more). It is perhaps the nature of this sluicing operation and the varying nature of the sludge from the desilting works that gives the apparently erratic changes in the median grain size of the river bed at Yuma, without any clear-cut coarsening of the bed sediment there.

While the writer believes that a coarsening of the bed material is not necessary to produce an increase in the roughness when the sediment load decreases, Drs. Einstein and Chien demonstrate by an example (their Table I) that a change in the grain size from 0.14 to 0.35 mm can make a significant change in the Manning "n" for the same discharge. It is presumed that the source of their data marked "measured" in Table I is Reference 10, as no new reference has been given. However, the discussors' use of 0.14 mm as the typical grain size before closure of Hoover Dam and 0.35 mm for after closure is not at all consistent with the observation of $D_{50} = 0.140$ mm on November 8, 1951 cited by them; in fact both $D_{50} = 0.14$ and $D_{50} = 0.35$ mm have occurred after closure. Thus, the discussors' deduction that the change of the roughness at Yuma from before closure to after is due entirely to a change in median grain size does not appear sound. The writer believes that the main contributing factor to the increase in the roughness is the great reduction in the bed material load whether the bed becomes slightly coarser or not.

SUMMARY

In general, this closure has: 1) attempted to explain the logic of uniqueness and nonuniqueness of various functional relationships; 2) answered several questions about procedure; 3) introduced more flume data in support of the writer's basic conclusions; 4) pointed out some of the difficulties in the analyses presented by the discussors; 5) given further opinions on field examples. For the reasons detailed above, the writer still adheres to all of the conclusions originally presented with the possible exception of No. 4(b).

The writer would like to express his deep appreciation for all of the efforts of the discussers in presenting so much valuable discussion on these controversial problems. Although all of the discussers have disagreed with all or part of the writer's conclusions, the writer regrets that, even after careful restudy of the available facts and of the discussions presented, he is unable to agree substantially with any of the discussers. Consequently, the closure has, of necessity, been a critical one. It is hoped that further information from both the laboratory and the field will resolve some of the problems brought up in this paper; in the meantime, it is hoped that this paper has served to point out the need for further research to improve our understanding of alluvial streams.

Discussion of
"RESEARCH NEEDS IN SEDIMENT HYDRAULICS"

by Enos J. Carlson and Carl R. Miller
(Proc. Paper 953)

ENOS J. CARLSON,[1] M. ASCE, and CARL R. MILLER,[2] A.M. ASCE.—The writers appreciate the discussions of their paper by Messrs. McCutchan and Shulits. Their discussions have pointed out some very interesting factors and they have added materially to the value of the paper.

The discussion by Arthur I. McCutchan points out the limitations of the USD-49 sampler in sampling a deep stream and using the imperial pint milk bottle. His situation where the Dawson River in Queensland rose to a height of 72 feet is certainly unusual. It would seem that a flood of this height would be very difficult to sample with the 50-pound D-49 sampler unless the velocity was comparatively low. The 100-pound P-46 would have certainly been more stable in the high velocities that probably occurred. The P-46 can be lowered to the bottom, the valve opened, then raised and thereby obtain an integrated sample in one direction only. This cannot be accomplished by the D-49 sampler. Therefore, the P-46 can sample twice the depth that can be sampled by the D-49.

Mr. McCutchan should be congratulated on using ingenuity and resources at hand to accomplish the job. Certainly in river hydraulics and sedimentation work, situations arise that are different from the usual standard problems with standard solutions. There has never been a flood of unusual proportions that has not taxed the ingenuity and resourcefulness of any engineer who is faced with the problem of obtaining as much data as possible for use in the design of projects.

Mr. McCutchan has referred to the necessity of obtaining adequate sediment measurements during flood flows. Determining the sediment transport during high runoff is essential for developing an adequate sediment rating curve. However, the actual contribution of the floods to the mean annual sediment load varies considerably for different rivers. In the determination of mean annual sediment load by the flow-duration, sediment-rating curve method, the answer is governed by the combination of the sediment-rating curve and the log-probability plot of the available stream flow records. For example, in Figure 5 of the writers' paper it can be noted that the bulk of the period sediment load is contributed by flows of 16,300 cfs or less. This discharge has a 3.5 percent chance of occurring during the period of record. Flows occurring this often would have a good chance of having an adequate number of sediment measurements to establish a curve. There are other streams where the extreme high flood flows contribute a major percentage of the mean annual sediment load. For example, on the Cheyenne River at

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Hot Springs, South Dakota, the bulk of the annual sediment load is contributed by flows in excess of 3,500 cfs which has a 1.0 percent chance of occurrence. Obtaining adequate sampling data in this case is difficult. In fact, the higher flows may not even occur in the period of sampling record available for the sediment study. It is therefore, evident that the importance of the flood flows should be established in the determination of the average annual long-time sediment load of the stream.

Mr. Shulits discussed many points which were certainly thought provoking and challenging to the hydraulic engineer interested in sediment engineering. Under the topic heading of "Bed Material Transport Computations," Mr. Shulits pointed out again, the number of formulas available for computing bedload transport, and the divergence of opinion as to which formula works best for a given set of conditions. It is not uncommon to find engineers using the formula that will given an answer that is reasonable and which is in the range that is desired. Some bedload formulas do give unreasonable results in certain situations.

Some comparisons of the results obtained by various bedload formulas, which will be helpful in evaluating the merits of each, are contained in the U.S. Geological Survey Report "Investigation of Sediment Transport, Middle Loup River at Dunning, Nebraska, 1956" which will be available as a published water supply paper in the near future. It is also gratifying to note that consistently good results are being obtained for total sediment load determination by the Modified Einstein Procedure.[3]

There has been a large amount of data obtained on fluvial morphology on many projects that have never been published, both on foreign projects and projects in this country. If those who are connected with this work or have the data would publish it, a great service would be performed for the hydraulic engineering profession. Maybe what is needed is to make it easy for those who have the data to have it published.

3. Application of Modified Einstein Procedure for Computation of Total Sediment Load—American Geophysical Union Transactions, Vol. 37, No. 2, April 1956.

Discussion of
"TRANSITION PROFILES IN NON-UNIFORM CHANNELS"

by Francis F. Escoffier
(Proc. Paper 1006)

L. J. TISON.—1. The author's study is of particular interest to the writer because a number of the writer's predecessors at the University of Gand and the writer himself have previously made studies in the same field. The great merit of the author lies in his having extended the theory in question to the field of non-uniform channels.

Apparently the publications of the writer's predecessors in the Chair of Hydraulics at the University of Gand did not receive a very wide distribution. For that reason the writer thought it might prove interesting to describe briefly some of the results obtained by them and by himself.

In these studies of water-surface profiles the practice was followed by taking the axis of the abscissa parallel to the bed of the channel and the axis of the ordinate perpendicular thereto. See Fig. 13. Under these conditions the equation for steady flow in a uniform channel, with a uniform distribution of velocities in the cross section, is written:

$$\frac{dy}{dx} = \frac{S_o - \frac{PQ^2}{C^2A^3}}{\sqrt{1 - S_o^2} - \frac{WQ^2}{gA^3}} \tag{52}$$

where P is the wetted perimeter and C is the Chezy coefficient. The numerator N on the right side of this equation vanishes for one or more values of y which values we shall write Yn; similarly the denominator D vanishes for a value of y which we shall write Y_c.

For a given discharge there exists a value for the bottom slope

$$S_o = S_p$$

such that

$$Y_n = Y_c = Y_p$$

We shall call this value the passage slope. The quantities S_p and Y_p are given by the two equations:

$$S_p = \frac{P_pQ^2}{C^2A_p^3} \tag{53}$$

and

1. Prof., Univ. of Gand, Belgium.

$$\sqrt{1 - S_p^2} = \frac{W_p Q^2}{g A_p^3} \tag{54}$$

from which we obtain

$$\frac{S_p}{\sqrt{1 - S_p^2}} = \frac{g\ P_p}{W_p C^2} \tag{55}$$

which is the equation for the passage slope. In fact one of eq's 53 and 54 has to be used with eq. 55 to determine S_p and Y_p for a given discharge and a given cross-section (in a uniform channel). For a uniform channel concave upward and for a constant value of C, S_p and Y_p have but a single value each. We can theoretically imagine C to vary in such a way that several values of S_p will be possible, but too little is known concerning the variation of C for small values of y and for circular conduits flowing nearly full, to justify the assertion that several values are really possible.

2. Horizontal elements, transition depths, and characteristic depths. These three different terms relate in fact to the same concept. The term horizontal element (element d'horizontalité) was introduced by Boudin, Professor at the University of Gand, in 1863. For each form of water-surface profile (called by him hydraulic axes) which he obtained Boudin sought to determine whether it presented a depth for which the tangent to the water-surface profile is horizontal. The condition to be satisfied is

$$\frac{S_o}{\sqrt{1 - S_o^2}} = \frac{dy}{dx} = \frac{S_o - \dfrac{PQ^2}{C^2 A^3}}{\sqrt{1 - S_o^2} - \dfrac{WQ^2}{gA^3}}$$

This condition leads easily to

$$\frac{S_o}{\sqrt{1 - S_o^2}} = \frac{g}{C^2} \left(\frac{P}{W}\right)_k \tag{56}$$

That is to say, a horizontal element exists for the depth y_k for which the quantity $\frac{P}{W}$ assumes the value $\left(\frac{P}{W}\right)$ k required by eq. 56.

This is always impossible for depths that lie between the critical depth Y_c and the normal depth Y_n because $\frac{dy}{dx}$ is negative in that region and a horizontal element corresponds to the positive value

$$\frac{dy}{dx} = \frac{S_o}{\sqrt{1 - S_o^2}}$$

Boudin at that time dealt separately with mild and steep bottom slopes. For mild slopes we have

$$\frac{S_o}{\sqrt{1 - S_o^2}} < \frac{\ }{C^2} \left(\frac{P}{W}\right)_{Y=Y_n} \tag{57}$$

It follows from eq. 5 that

$$\frac{g}{C^2}\left(\frac{P}{W}\right)_k < \frac{g}{C^2}\left(\frac{P}{W}\right)_{Y=Y_n} \tag{58}$$

and as $\frac{P}{W}$ increases in value with Y (for cross sections that are concave upward or circular), Boudin, assuming C to be constant, concluded that

$$Y_k < Y_n$$

that is to say a horizontal element can exist only for depths that are less than Y_n and consequently also less than Y_c.

For steep bottom slopes Boudin showed by similar reasoning that a horizontal element can exist only for depths that are greater than Y_n and consequently greater than Y_c.

Boudin did not seek to find the conditions under which horizontal elements occur in the foregoing cases. The writer has shown in his "Cours d'Hydraulique" (2) that in mild-slope channels (in which C is constant) a transition slope or transition depth will not occur if

$$\frac{S_o}{\sqrt{1 - S_o^2}} < \left(\frac{P}{W}\right)_o \tag{59}$$

where $\frac{P}{W_o}$ is the value of $\frac{P}{W}$ for y = o. We see in effect that since $\frac{P}{W}$ is an increasing function of y it is impossible to satisfy eq. 56 if inequality 59 holds true.

Similarly, a horizontal element is impossible in a steep-slope channel if

$$\frac{S_o}{\sqrt{1 - S_o^2}} > \frac{g}{C^2}\left(\frac{P}{W}\right)_\infty$$

Boudin also stated that in the case of a passage slope a horizontal element occurs for

$$Y_n = Y_c = Y_p$$

The writer will return to this point.

3. Boudin's development was completed by Merten in his memoir of 1906 (3). Merten clearly established a number of points in regard to passage slopes and stopped with the consideration of inflexion points, a matter closely related to that of transition depths. He established notably the possibility of such points in mild-slope channels (Boudin had recognized their existence for steep-slope channels).

4. The writer took up the question of horizontal elements in his "Cours d'Hydraulique," with the significant change of introducing a varying value of C. This coefficient in reality is not a constant but a function that increases with y. As a consequence, for small values of y the function $\frac{gP}{C^2W}$ is not necessarily a diminishing function of y, but presents an appearance like that

shown in Fig. 14. It follows that we obtain 0, 1, or 2 horizontal elements (transition depths) accordingly as $\frac{S_o}{\sqrt{1 - S_o^2}}$ is less than, equal to, or greater than $\left(\frac{gP}{C^2W}\right)$ min. This result is equally applicable to cross sections that are concave upward and to circular cross sections.

We should always keep in mind the fact that the study of horizontal elements is, in practice, greatly complicated by the fact that the variation of C with y is not well understood for small values of y and for values of y approaching the diameter in a circular section. Tests carried out by Wildox (4) show, in the latter case, that the theoretical variation of the normal discharge with the depth of water does not correspond to reality.

5. The passage slope.

As the writer has already stated Boudin was of the opinion that since the condition for the occurrence of a horizontal element is

$$\frac{S_o}{\sqrt{1 - S_o^2}} = \frac{g}{C^2}\left(\frac{P}{W}\right)_k$$

and furthermore that in the case $S_o = S_p$

$$\frac{S_p}{\sqrt{1 - S_p^2}} = \frac{g}{C^2}\left(\frac{P}{W}\right)_p$$

the tangent to the water-surface profile should be horizontal for a depth equal to the depth of passage (i.e., in the author's terminology for $Qn = Qc = Q$).

This result is nevertheless not general. In effect, eq. 1

$$\frac{dy}{dx} = \frac{N}{D},$$

yields for $y = Y_p$

$$\frac{dy}{dx} = \frac{o}{o}.$$

The application of the rule of the Marquis de l'Hospital yields:

$$\frac{dy}{dx} = \frac{N'}{D'}$$

and the tangent to the water-surface profile for $y = Y_p$ will be horizontal if

$$\frac{S_o}{\sqrt{1 - S_o^2}} = \frac{N'}{D'}$$

(This is equivalent to

$$\frac{dQn}{dQc} = 1$$

in the author's terminology).

The foregoing equation is not always satisfied. Thus in a rectangular channel having a width λ it can be shown that

$$\frac{S_p}{\sqrt{1 - S_p^2}} > \frac{dy}{dx} \; Y_p$$

It is therefore clear in that case that a horizontal element does not occur. However, if λ approaches infinity we see that the tangent approaches a horizontal position.

6. An additional remark in regard to passage slopes.

The normal-depth and critical-depth profiles divide the space above the bottom of the channel into three regions. In the highest and the lowest of these regions the sign of $\frac{dy}{dx}$ is positive but in the intermediate region it is negative. A bottom slope equal to the passage slope causes the intermediate region to vanish, and as a consequence $\frac{dy}{dx}$ must always be positive. All of this relates obviously to a uniform channel concave upward). As a result it follows that in this case a water-surface profile can pass through the depth Y_p only with diminishing depths. This conclusion can be considered as a logical consequence of another property of uniform channels that are concave upward, i.e., that in such channels a chute (abrupt passage from a depth greater than Yc to one less than Yc) is impossible, whereas a jump, the inverse phenomenon, is possible.

By contrast, in the case of a circular section, the existence of a passage slope is accompanied by a region in which $\frac{dy}{dx}$ takes on negative values so that a water-surface profile with diminishing depths is possible.

7. Non-uniform channels.

Without wishing to detract from the author's valuable work in this field the writer would like to indicate another method for the study of transition profiles in non-uniform channels, a method that follows logically from the foregoing discussion.

For a non-uniform channel eq. 1 becomes

$$\frac{dy}{dx} = \frac{S_o - \dfrac{PQ^2}{A^3C^2} + \dfrac{Q^2}{gA}\dfrac{\partial A}{\partial x}}{\sqrt{1 + S_o^2} - \dfrac{Q^2W}{gA^3}} \tag{60}$$

A horizontal element corresponds to the condition

$$\frac{S_o}{\sqrt{1 - S_o^2}} = \frac{\dfrac{PQ^2}{A^3C^2} - \dfrac{Q^2}{gA^3}\dfrac{\partial A}{\partial x}}{\dfrac{Q^2}{g}\dfrac{W}{A^3}} \tag{61}$$

or

$$\frac{S_o}{\sqrt{1 - S_o^2}} = \frac{gP}{C^2W} - \frac{1}{W}\frac{\partial A}{\partial x} \tag{62}$$

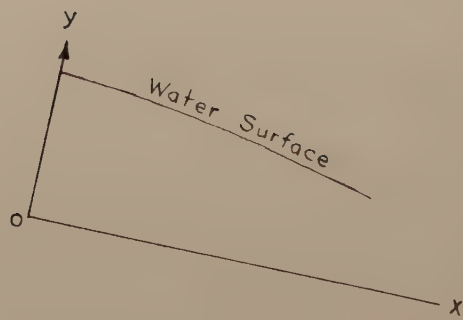
y
Water Surface
o
x

Fig. 13

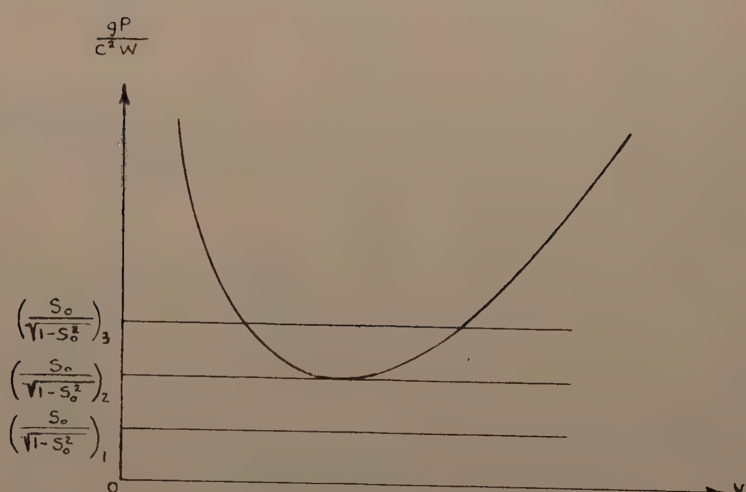
$\frac{gP}{c^2 W}$
$\left(\frac{S_o}{\sqrt{1-S_o^2}}\right)_3$
$\left(\frac{S_o}{\sqrt{1-S_o^2}}\right)_2$
$\left(\frac{S_o}{\sqrt{1-S_o^2}}\right)_1$
o
y

Fig. 14

This result can be used to investigate channels with simple cross sections (rectangular for example).

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FRANCIS F. ESCOFFIER,[1] A.M. ASCE.—It appears that varied terminology exists in regard to the depth which the writer has called a "transition depth." In the studies by Mouret and Lazard this depth is called the "characteristic" depth (hauteur caractéristique). In the studies by Boudin, Merten, and Tison a distinction is made between the case when the tangent to the water-surface profile is horizontal and when it is not. In the first case we have either

$$Q_n = Q_c \neq Q$$

or

$$Q_n = Q_c \quad \text{and} \quad \frac{dQ_n}{dQ_c} = 1$$

and the segment of the water-surface profile passing through that depth is called a horizontal element (élément d'horizontalité) and although no specific name is assigned to this depth it is identified by the subscript n, i.e.,

$$y = Y_n$$

In the second case we have

$$Q_n = Q_c = Q$$

and the depth in question is called a passage depth (hauteur de passage). It appears to the writer that since for a given channel (with a given bottom slope) the depth in either case is the same the term passage depth could be used in both cases.

Mr. Lazard has expressed doubt as to the reality of some of the water-surface profiles obtained by Mouret and the writer. The writer would like to point out that his profiles, and no doubt those of Mouret also, were deduced logically from the dynamic equation in general use for flow in open channels, (eq. 5 in the writer's paper) and are therefore as valid as that equation. Possibly the most significant source of error arises from the neglect of the

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curvature of filaments. For that reason the profiles become completely unreliable in the vicinity of a passage through critical depth if the equation yields a vertical water surface.

The importance of the velocity-distribution coefficient which has been stressed by Mr. Lazard, is conceded by the writer. However, it should be pointed out that the neglect of this coefficient is not ordinarily as great a source of error in non-uniform channels as it is in uniform ones. We can introduce this coefficient into our calculations by rewriting eq's 50 and 51 as follows:

$$F' = \frac{\alpha}{2gA^2} + \frac{L}{2K^2} \tag{50'}$$

$$F'' = \frac{\alpha}{2gA^2} + \frac{L}{2K^2} \tag{51'}$$

The writer shares with Mr. Lazard the wish that laboratories interest themselves in the various water-surface profiles that arise from the theory of transition profiles and undertake to study them in models.

The writer was interested in learning of the work done by Boudin, Merten, and Tison on the subject of transition depths and regrets that his own research before the writing of his paper failed to disclose their literature. The early date of Boudin's work (1863) is to be noted.

Mr. Lazard has suggested that the different types of water-surface profiles that arise in the theory of transition profiles be named or designated in some way. Unfortunately the number of such types is so great that the list of names would be too long to be useful. However, the writer feels that it would be helpful to distinguish four types of transitions. These are illustrated by means of the saddleback in Fig. 15. The four terms and their counterparts in Fig. 15 are as follows:

Transitional minimum	EFG
Transitional maximum	HIJ
Transitional drop	AOB
Transitional rise	COD

The first two may be called "non-singular" transitions because F and I, the points of intersection with the transition profile, are not singular points. On the other hand, the last two may be called "singular" transitions because they pass through the singular point O.

Prof. Tison's remark in regard to the possibility of a water-surface profile passing with diminishing depths through a transition profile (in a uniform channel) were made with Fig. 2 in mind. This figure, which is the result of carelessness on the part of the writer, is quite misleading. It really corresponds to the condition

$$o > \frac{dQ_n}{dQ_c}$$

rather than simply

$$1 > \frac{dQ_n}{dQ_c}$$

and represents a type of water-surface profile that can occur only in a closed conduit that is flowing nearly full. Figs. 2a and 2b which supersede Fig. 2 were prepared to clarify this matter. The water-surface profiles in both Fig. 1 and Fig. 2a pass from rapid to tranquil flow whereas the one in Fig. 2b passes from tranquil to rapid flow.

Prof. Tison's eq's. 60, 61, and 62 provide us with another method for investigating the properties of transition profiles. They include the factor

$$\sqrt{1 - S_0^2}$$

to correct for the fact that the channel cross sections are not vertical. The importance of this correction increases with the slope of the channel.

The writer wishes to thank Messrs. Lazard and Tison for their most interesting discussions.

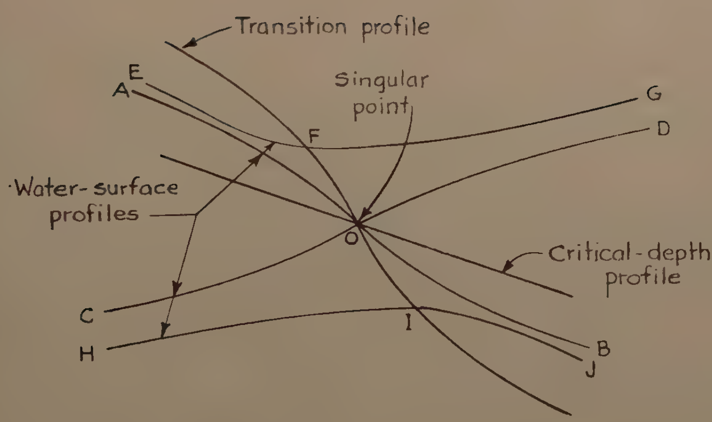

Fig. 15

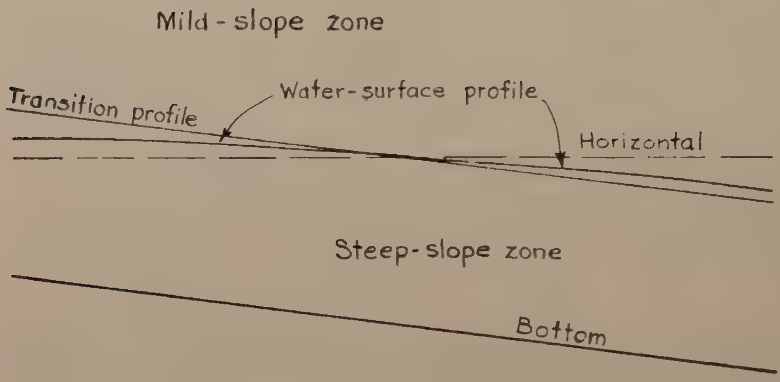

Fig. 2a

Fig. 2a Transition profile in uniform channel $\left(1 > \frac{dQn}{dQc} > 0\right)$

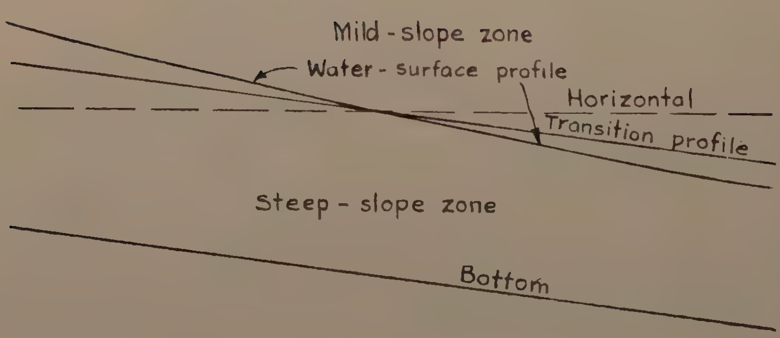

Fig. 2b

Fig. 2b. Transition profile in uniform channel $\left(0 > \frac{dQn}{dQc}\right)$

Discussion of
"SEVEN EXPLORATORY STUDIES IN HYDRAULICS"

by Hunter Rouse
(Proc. Paper 1038)

M. B. McPHERSON,[1] A.M. ASCE., and R. G. DITTIG[2].—These exploratory studies in hydraulics are noteworthy contributions to the profession. The authors of the various sections and the Iowa Institute of Hydraulic Research are to be commended for accomplishing so much in so little time. The section "Surface Profiles at a Submerged Overfall" was stated to be of an exploratory nature, nevertheless it contributes a valuable extension to the range of previously published information. The objective of this discussion is to introduce some of the possible limitations or differences inherent in the authors' study, and to extend the data for the case of a Froude number of one.

"Hydraulic Jump" and "Riding Nappe" Regimes

Previous studies of the hydraulic jump at an abrupt drop[3] have included a maximum z/y_o of 4, wherein z is the vertical distance between the upstream and downstream channel floors. The z/y_o of the authors' study is 6.67. In the reference cited, Eq. (49b) was applied to the case where the toe of a roller-type jump is just upstream of the drop. This corresponds to the highest profiles of the authors' Fig. 8 for F = 2, 3 and 4, and to the "hydraulic jump" demarcation curves in their Fig. 10. Equation (49a) was applied to the case where the toe of the roller-type jump is just downstream from the drop. According to Fig. 8, it does not appear that this case occurred in the authors' study. Between these extremes, the reference states that a "standing wave" will be obtained. In Fig. 8 the profiles for F of 3 and 4, and corresponding y_t/y_o of +2 and +3 appear to have the configuration of a "standing wave." In Fig. 10 this is called a "riding nappe." Further, according to the trend of the curves in Fig. 61 of the reference,[3] for F up to 4, one would expect the authors' data to agree only with Eq. (49b):

	Values of y_t/y_o		
	F = 2	F = 3	F = 4
Eq. (49b) Toe of roller-type jump just upstream of drop	2.8	4.2	5.6
Fig. 10—Average of "rising" and "falling" tailwater	2.0	2.8	3.9
Eq. (49a) Toe of roller-type jump just downstream of drop	1.4	2.0	2.7

1. Associate Prof. of Civ. Eng., Lehigh University, Bethlehem, Pa.
2. Asst. Prof. of Civ. Eng., Lehigh University, Bethlehem, Pa.
3. "Design of Channel Expansions," by Hunter Rouse, B. V. Bhoota and En-Yun Hsu, Trans. ASCE, Vol. 116 (1951), p. 360.

The above values for Eq. (49b) are within the "hydraulic jump" regime of Fig. 10, and those for Eq. (49a) are within the "riding nappe" regime. It would appear, therefore, that the regime characteristics for z/y_o of 6.67 differ from those previously obtained for a range of z/y_o of 1 to 4.

"Plunging Nappe" Regime

According to Moore,[4] a "plunging nappe" (i.e., "submergence" of a jump located in the downstream channel) should be expected for a ventilated lower nappe and Froude number of 1, so long as y_t/z is not less than -0.61. In addition, for y_t/y_o = -0.02, and with a ventilated lower nappe and F = 1, the depth of water under the lower nappe was only 0.69z; and a depth equal to z was not obtained until y_t/y_o = +0.68[5]. In the lowest photo of Fig. 9 the space under the lower nappe appears to be filled with water. It is assumed that the data of the authors' study are for a non-ventilated lower nappe, and that the space beneath the lower nappe was always filled with water.

Evaluation of the brink depth for F = 1 to F = 10.6, with a ventilated lower nappe, has been presented by Rouse.[6] Did the authors' profile data for a "plunging nappe" include measurements of the brink depth?

Application to Broad Crested Weirs (Calculated F equal to unity)

Diversion structures, such as cofferdams and cooling-water intake dams, are often constructed of sheet piling and have a horizontal crest and square edges. In the absence of adequate stream bed protection below such a structure a "plunging nappe" would seriously erode any tractable downstream material. The authors' study demonstrates the fact that a tailwater level in the vicinity of the crest is needed to deflect the main flow away from the downstream channel floor.

The remainder of this discussion pertains to weirs with proportionately narrow crests, where the headwater above the crest, H, is greater than about three-tenths the breadth of the weir, L (see Fig. 1). For the case of a "plunging nappe" and a constant H, a decrease in the weir breadth will result in a steeper water surface profile over the crest, and the flow entering the tailwater should also assume a steeper angle. One would therefore expect a narrow weir with a given z/y_o to exhibit at least two different effects: (1) as the H/L ratio increases, the y_t/y_o ratio required to avoid a "plunging nappe" should increase, and (2) a small surface roller, or hydraulic jump, should be obtainable despite the fact that the computed F is approximately unity.

In conjunction with this discussion tests were performed in a 1.50 ft. wide flume on seven different weirs, with L varying from 0.43 to 1.00 ft. and z varying from 0.28 to 1.00 ft. No ventilation was provided, and the space under the lower nappe was filled with water for all data reported. With a

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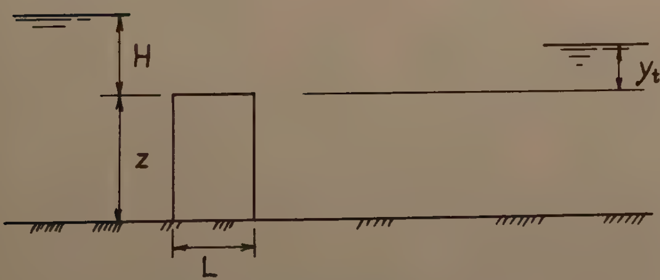
4. "Energy Loss at the Base of a Free Overfall," by Walter L. Moore, Trans. ASCE, Vol. 108 (1943), p. 1358.
5. "Flow geometry at Straight Drop Spillways," by Walter Rand, ASCE Proc. Sep. No. 791, Sept. 1955, p. 791-5.
6. "Discussion of Energy Loss at the Base of a Free Overfall," by Hunter Rouse, Trans. ASCE, Vol. 108 (1943), p. 1386.

TABLE 1 - Summary of Weir Test Results

z/y_o	H/L	Values of y_t/y_o (1)	(2)	(3)	(4)
2.00	0.35	1.05	0.75	1.58	1.60
"	0.54	0.92	0.76	- -	- -
"	0.78	0.96	0.78	1.61	- -
3.00	0.37	0.76	0.37	1.57	1.53
"	0.55	0.79	0.50	1.54	1.47
"	0.77	0.80	0.38	- -	- -
4.00	0.28	0.60	0.04	1.55	1.39
"	0.41	0.61	0.10	1.51	1.29
"	0.62	0.67	-0.04	1.48	- -
"	0.78	0.68	0.03	- -	- -
5.00	0.38	0.54	-0.13	1.51	1.35
"	0.50	0.45	-0.18	1.44	- -
"	0.76	0.65	0.06	1.53	- -
6.67	0.16	0.20	-0.19	1.52	1.45
"	0.23	0.19	-0.46	1.58	1.23
"	0.38	0.42	-0.32	1.33	1.20

(1) Rising tailwater - riding nappe starts,
(2) Falling tailwater - plunging nappe starts,
(3) Rising tailwater - small roller forms on crest,
(4) Falling tailwater - small roller on crest ceases.



H = measured head above weir crest

y_t = measured tailwater relative to weir crest

$y_o = \sqrt[3]{\frac{q^2}{g}}$, where q = measured discharge, cfs/ft

Fig. 1 - Definition Sketch - Broad Crested Weirs (Square corners, horizontal crest)

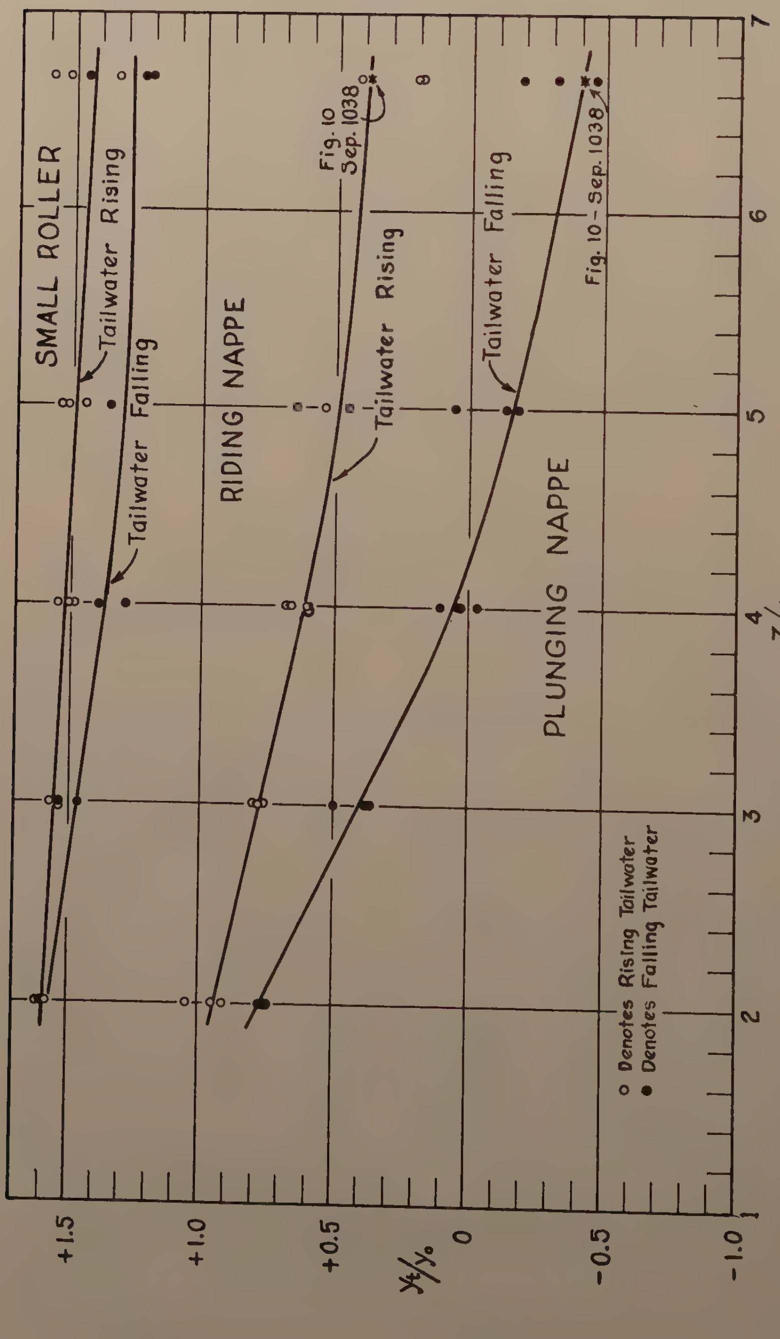


Fig. 2 - Regime Demarcation Lines - Broad Crested Weirs
(Space under lower nappe filled with water)

rising tailwater, the following phases were noted in the order given: (a) plunging nappe; (b) riding roller (within a very short range of tailwater); (c) riding nappe (standing wave)—well downstream from the crest; (d) flat tailwater—over the weir crest and for a considerable distance downstream; (e) riding nappe (standing wave)—immediately downstream from the crest; (f) feeble roller-on, or immediately adjacent to, the crest.

With a falling tailwater the same phases were noted, but in the reverse order. For the lower H/L values each of the above phases were more clearly defined than at the higher H/L values (greater than about 0.5).

The test results are summarized in Table 1, and are plotted in Fig. 2. The lines of demarcation between a plunging nappe and a riding nappe appear to be independent of the magnitude of H/L, for the ranges tested. This is contrary to what was expected. It should be noted that there is reasonably good agreement with the two points for F = 1 from Fig. 10 of the authors' paper. For a z/y_o of 6.67 only small values of H/L were attainable with the equipment available. The results obtained give some indication of the displacement which might be expected in the lower curves of Fig. 10 for a range of z/y_o from 2 to 5.

The uppermost curve in Fig. 2 is for the occurrence of the feeble roller on, or immediately adjacent to, the crest. The companion curve indicates the termination of that roller for a falling tailwater. It was found that the onset of submergence (where submergence is characterized by a continuously rising headwater with a rising tailwater) coincided closely with the occurrence of the roller. In the range of H/L from 0.3 to 0.5 submergence occurred at the same time that the roller formed. For an H/L of about 0.6 submergence immediately preceded the formation of the roller. For the highest H/L value attained (approximately 0.8) submergence began in the lower riding nappe regime but was of no practical significance until the roller was formed. Therefore, the uppermost curve of Fig. 2 can be regarded as a nominal lower limit of submergence for the case of a rising tailwater.

F. PADERI[1].—Owing to an interest of long standing, the writer joins in the discussion on the seventh study, "Influence of Slope and Roughness on the Free Overfall," by J. W. Delleur, J. C. I. Dooge, and K. W. Gent (E. M. Laursen, Adviser).

This interest began with the classic publications by Boss(1) and Rouse(2) (1933) concerning a channel with horizontal bottom and is illustrated by what was reported in the Bulletin, "Hydraulic Research," edited by the International Association for Hydraulic Research, Delft, Pays-Bas, Volume 9, for the year, 1953 (page 195) and Volume 10 for the year, 1954 (page 191). The importance of this subject, which concerns a region of the channel which may be said to be crucial—as it is where various hydraulic phenomena occur (backwater, free overfall, transformation of energy from one form into another, energy dissipation, etc.)—justifies the interest. Further investigations on the above phenomena show that there is considerable scientific and technical application.

In 1952 the writer resumed (with the highly valuable contribution of A.N.I.D.E.L.) his own theoretical (with the application of the momentum and continuity principle) and experimental studies in a wooden, freely tilting channel, 6 meters long, 0.20 meters wide, and 0.20 meters deep. Since then

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he has performed at intervals, in the Laboratory of Hydraulic Institute of the University of Pisa, experiments in a glass-walled tilting flume, 9 meters long, 0.50 meters wide, 0.50 meters deep, and with unitarian discharges up to 0.2000 cubic meters per second per meter. These values agree closely with the flume dimensions and the discharge values investigated by the authors.

Of the experiments performed by the writer on the horizontal bottom and the positive and negative slopes with slopes $\underline{i}$ from -0.003 (adverse slope) to +0.0055, results and first analytic and graphic elaboration concerning the horizontal bottom(3) and the mild slopes cases(4) have been published. The experimental investigation referring to more than 15 profiles was also extended upstream of the critical section, and the discharges were always measured by the volumetric method.

Because of their peculiarity, the writer has preferred to separate the experimental results concerning the mild slopes from those concerning the steep slopes, according to discharge and water depth. With additional data already determined, the elaboration of experimental results proceeds.

In the aforementioned report by the authors, the writer has found particularly interesting the application of the momentum principle because of its relative simplicity as the result of experiments. This application included the introduction of factors, K_1, K_2, etc., which are ratios of real and reference (opportunely selected) quantities. In this, as in other branches of hydraulics, a remarkable contribution to the practical resolution of the problem under examination can be had by the introduction of the factor K when the value of the said factor can be easily determined by the experimental or theoretical methods. (A particularly favorable case occurs when the K factor is nearly constant in the zone under consideration.)

The writer wishes to state that this contribution by the authors and those by others should enlarge our knowledge of this important and complex subject on either the theoretical side or the experimental side. This will allow more and more profitable results with regard to technical applications.

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2. Rouse, H. "Verteilung der Hydraulischen Energie bei einem lotrechten Absturz," Berlin, Oldenbourg 1933.

3. Paderi, F. "Sulla chiamata di sbocco, Notizie preliminari per canale a fondo orizzontale." L'Energia Elettrica No. 10 ottobre, Milano 1954.

4. Paderi, F. "Sulla chiamata di sbocco in canale a fondo declive," L'Energia Elettrica, N. 8 agosto, Milano 1956.

Discussion of
"FLOOD PROTECTION OF CANALS BY LATERAL SPILLWAYS"

by Harald Tults
(Proc. Paper 1077)

VEN TE CHOW,[1] A.M. ASCE.—The writer is interested in the hydraulic treatment on the nature of flow along the lateral spillway. Here is an example to show how the energy principle as well as the momentum principle can be applied to a hydraulic problem and both give the same result. In fact, both principles should produce the same result.

For a spacially varied flow, there is a need to distinguish the problem of the flow with an increasing flow from that with a decreasing flow. The lateral spillway as treated by the author belongs to the latter type. An example of the former type is the lateral-spillway channel.

In the case of a lateral-spillway channel that is designed for a spacially gradually varied flow with increasing discharge, either the energy principle or the momentum principle can be used. In solving such a problem, however, the total momentum is generally assumed unchanged whereas the energy content will be affected by the added water. Therefore, the use of the momentum principle is more simple and suitable. In other words:

(a) The problem can be solved by the direct application of the momentum principle.

(b) The problem can be solved by the energy principle but there is a need for the correction for the energy brought in by the added water. In deriving a differential equation for the flow profile, the energy due to the added discharge dQ per elementary distance dx should be added to the total energy along the length of the channel during the time interval dt. This kinetic energy per pound of water is equal to (mass)(velocity)^2/g (unit weight of water)(volume) = $(w\,dQ\,dt)(\alpha V^2)/gwA\,dx = \alpha V dQ/gA$, in which w is the unit weight of water, α the energy correction coefficient, V the mean velocity of flow, A the water area of the flow, and g the gravitational acceleration. Thus, by adding this term to the general Bernoulli equation, the result will be the same as that produced by the momentum principle.

In using either (a) the momentum principle or (b) the energy principle, the differential equation of the flow profile in prismatic channels may be expressed by

$$\frac{dy}{dx} = \frac{S_o - S - 2\alpha Qq/gA^2}{1 - \alpha Q^2/gA^2D} \qquad \text{(D-1)}$$

in which S_o is the bottom slope, S the friction slope determined by an energy formula, q the increased discharge per unit length of the channel, and D the hydraulic depth defined as the water area divided by the top width. When the momentum principle is applied, a momentum coefficient β should have

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been used instead of α, provided S is evaluated by a formula also based on the momentum principle. Since S is computed by an energy formula, it is correct to use α.

For the convenience of numerical integration, Eq. D-1 can be converted to an expression used by Hinds(12) for the drop in water surface:

$$\Delta y = \frac{\alpha Q_1 (V_1 + V_2)}{g(Q_1 + Q_2)} (\Delta V + \frac{V_2}{Q_1} \Delta Q) + S \Delta x \tag{D-2}$$

In the case of a lateral spillway as described by the author, the total energy is assumed unchanged whereas the momentum will be changed due to the diversion of water. Therefore, the energy principle will be found much simpler and suitable. As demonstrated by the author:

(a) The problem can be solved directly by the energy principle.

(b) The problem can be solved by the momentum principle, but there is a need for the correction for the momentum being carried away by the diverted water. Consequently, the momentum expressed by Eq. (4) should be subtracted from, instead of "added" to, the total momentum difference by Eq. (3).

The use of either the energy principle or the momentum principle will result in a differential equation of the flow profile for a spacially gradually varied flow with decreasing discharge as follows:

$$\frac{dy}{dx} = \frac{S_o - S - \alpha Qq/gA^2}{1 - \alpha Q^2/gA^2 D} \tag{D-3}$$

It is interesting to note that the difference between Eqs. D-1 and D-3 lies only in the coefficient of the third term in the numerator of the right-hand side of the equations.

For the convenience of numerical integration, the writer has derived an equation similar to Eq. D-2 but for flow of decreasing discharge:

$$\Delta y = \frac{\alpha Q_1 (V_1 + V_2) \Delta V}{g(Q_1 + Q_2)} (1 - \frac{\Delta Q}{2Q_1}) + S \Delta x \tag{D-4}$$

which should agree with Eq. 6 obtained by the author.

The application of either the energy principle or the momentum principle to the spacially gradually varied flow depends on the assumption and condition of the problem. For example, the overland flow on a flat surface due to rainfall is a spacially varied flow of increasing discharge. In accordance with the previous discussion, Eq. D-1 should apply. However, for the practical reason of ignoring the raindrop-momentum, an equation belonging to the type of Eq. D-3 has been developed and used in analyses.[2]

The author is to be complimented for collecting and supplying ample information on the design of lateral spillways for the benefit of practicing engineers.

2. "The Surface-Profile of Overland Flow," by C. F. Izzard, Trans. Amer. Geophys. Union, Pt. VI, pp. 959-968, 1944.

Corrections:

p. 1077-2, footnote 2 should be "Mitteilungen aus dem Dresdner Flussbaulaboratorium," a/Forschungsarbeiten auf dem Gebiete des Ingenieurwesens, Nos. 200 and 201, Berlin, 55 pp, 1917 . . . etc.

p. 1077-3, the first negative sign on the right-hand side of Eq. (3) should be positive.

Discussion of
"THE PROBLEM OF RESERVOIR CAPACITY
FOR LONG-TERM STORAGE"

by A. Fathy and Aly S. Shukry
(Proc. Paper 1082)

MIKHAIL S. HANNA[1].—In the writer's view, the paper by Messrs. Fathy nd Shukry on the problem of long-term storage gives a very lucid account of he fundamental factors in that problem. There are, however, a few points to hich he would like to draw attention.

The first point is that, despite the difference in the lines of approach folowed, there is a very close similarity between the expression of the ideal ange arrived at the authors in Eq. (18) and Dr. Hurst's formula (5). According to the authors,

$$R = c\ M_a\ \left(\frac{N}{2}\right)^{1-m} \qquad (18)$$

nd according to Hurst,

$$R = \sigma \left(\frac{N}{2}\right)^{k} \qquad (5)$$

Thus the standard deviation (σ), which appears in (5), is replaced in (18) y the quantity ($c\ M_a$) which, by the authors' relation (16), should be the aximum negative deviation of one observation. Furthermore, the index k n (5), to which Dr. Hurst assigns the universal value 0.72, is replaced by a ariable index (1-m), wherein the value of m has to be determined for each henomenon independently. The value given to k by Dr. Hurst amounts to xing the value of m at 0.28.

The second point concerns the questions of the correction for sub-annual uctuations in the supply or the demand and flood-relief storage. In speaking bout the former question, the authors have stated that in the majority of ases the need for a correction may be eliminated by a certain choice of the ate at which the hydraulic year is supposed to begin. That statement needs ome clarification. The cases to which it applies are those where floodorage is provided in full, i.e., without automatic spilling. The proposed igh-level reservoir at Aswan belongs to that class. The principles governng design in such a case may be briefly demonstrated as follows:

Let us suppose, for the sake of simplicity, that the draft remains constant roughout the year and that there is a sub-annual variation in the supply ith a low stage from the first of January to the end of June and a high stage uring the remaining six months.

Supposing further that the supply also remains constant during each of its o stages, the correct storage capacity needed to guarantee a given draft

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would be directly obtainable by a half-yearly account. It is clear that an account based on a time-unit during which both the supply and the draft remain constant would not need any correction for sub-annual fluctuations.

Now, let the cumulated departures of the half-yearly observations from their mean for a certain series of years be as shown by the solid line in Fig. A. The maximum deficit within that period will be the range between the two points a and b.

If the time-unit were taken as one solar year, the plot of the cumulated departures would be as shown by the dash-dot line, and the maximum deficit would be the range between the points a and d. That range is smaller than the true range by an amount de.

If the time-unit were taken as one hydraulic year (beginning with the high stage in the supply) the plot of the cumulated departures would appear as shown by the dashed line, and the maximum deficit would be given by the points c and b. The range between these two points is smaller than the true range by an amount cf.

It is clear that in the second case sub-annual and flood storages become virtually one and the same thing. Since flood storage would, as a rule, be determined with respect to the highest anticipated flood, the extra capacity provided should cover the correction for sub-annual fluctuation that may be needed in any year.

There are, however, certain conditions to be satisfied if flood regulation is to be carried out successfully. It is obvious that the total supply of the year preceding the point where the deficit period begins according to the annual account (point c in Fig. A) must be greater than or, at least, equal to the total requirements. If it were less the beginning of the deficit period would be shifted back. Thus, in order not to spill any water unnecessarily, regulation in that year should be so arranged that a volume at least equal to the deficit in the supply below the requirements in the low stage be retained in the reservoir at the date on which that deficit begins. Thus spilling would be confined to the actual excess of the total supply of the year over the total requirements. At the same time, that regulation should be such that neither the prescribed maximum storage nor the maximum permissible escapage discharge (which is fixed with regard to conditions in the downstream channe be exceeded.

The application of such a regulation to a given set of observations is easy In practice, however, we should have to begin regulation in any year with little or no advance knowledge of the states of either the high stage or the lo stage of the natural supply. Forecasts of that supply would be needed for the longest possible period ahead, and suitable sliding scales would have to be applied to fulfil the required conditions as closely as possible.

The third point to which the writer would like to draw attention concerns the computation of the capacity needed in the Aswan High-Dam scheme. An account of the hydrological computations on which the official design is base has been published by the Sadd El-Aali (High Dam) Authority in 1955. A discussion of the design itself would be out of place herein but a brief review of the methods used in the computation may be useful on the present occasion.

The computation is based on Dr. Hurst's "century storage" theory, where the ideal range is first determined for a working period of 100 years with th aid of the universal formula (6). The storage needed to guarantee a given draft less than the mean is then calculated as a certain fraction of that rang with the aid of an empirical rule such as that given in Eq. (8). In the report

nder review, a new rule was adopted in the form:

$$\log_{10}\left(\frac{S}{R}\right) = -0.08 - 1.05\left(\frac{M-D}{\sigma}\right) \qquad (27)$$

vhere D is the draft, which must obviously include storage losses.

In working out R, the mean and the standard deviation were taken from the xisting record for 84 years. The values found were 93 and 19.8 nilld.m^3/annum, respectively, for the last two and 331 milliards for the ange.[2] That computation, however, was followed by the remark:

"It is to be noted that this value (of the range) is lower than would be found from the past record of Aswan alone since this gave an unusually high value of R. However, the equation (6) gives the most likely value for the future."

Perhaps not everybody would agree to that statement. In the writer's pinion, a record of 84 years duration would give a far more likely value for he future than an empirical universal rule.

Apart from that, there is another source of uncertainty in the choice of 00 years in particular as the length of the working period. It is evident that f we took a working period of 50 or 150 years, the range and all the figures lerived therefrom would be considerably altered.

Anyhow, the next step taken was to compute the draft corresponding to a torage of 70 millds. By (27) that draft was found to be 82 millds./annum. n adopting a working draft of 70, the margin of 12 was presumably left to over storage losses. The treatment of losses as an overhead allowance onstitutes a fundamental difference between the method used and that sug-ested by Messrs. Fathy and Shukry.

Next a computation was given to justify the choice of a capacity of 70 nillds. through consideration of the increments of storage losses corres-onding to successive increments in the draft above 82 millds./ann. It was hown that the gain in effective draft corresponding to an increase of capacity rom 70 to 110 millds. would be only about one milliard. That result, how-ver, is suggestive of the possibility that the capacity of 70 millds. itself night not be economical. Taking a capacity of 45 millds., for example, the ross draft according to (27) would be 78.2 millds./annum. At that capacity e actual volume in the reservoir would be 75 milliards (including the dead torage of 30 millds.). The corresponding reservoir area minus natural iver area under the reservoir[3] is 2660 km^2. Assuming a rate of loss per ear equivalent to a depth of 3 metres on the water surface, the annual loss t full storage level would be 8 millds. Hence the effective draft would be, t least,

$$78.2 - 8.0 = 70.2 \text{ millds./ann.}$$

Now, with a working capacity of 70 millds. the actual volume in the reser-oir would be 100 millds. The excess reservoir area at that volume is

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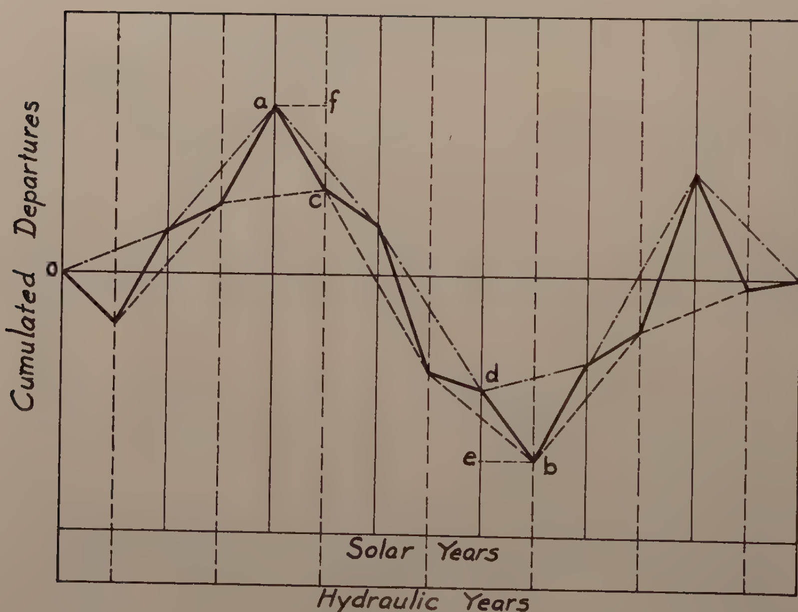
. It may be noted that the standard deviation is much lower than the observed maximum negative deviation in the supply of one year, which is 51 millds./ann.

That deduction is made because the natural supply, as observed at Aswan, already includes losses from the river surface.



Cumulated Departures
0
a
f
c
d
e
b
0
Solar Years
Hydraulic Years

Fig. A

3300 km^2 and the corresponding loss is 9.9 millds./annum. The gross draft according to (27) works out at 81.8 millds./annum. Hence the effective draft would be

$$81.8 - 9.9 = 71.9 \text{ millds./ann.}$$

Thus, the gain in effective draft for an increase of 25 milliards in the capacity would be only 1.7 millds./annum.

The optimum capacity should, of course, be determined with reference not to capacity-yield increments but to cost-yield increments. It does not seem that the latter consideration was taken into account.

The foregoing observations are put forward solely in order to stress the need for a final liquidation of the long-term storage problem towards which a good deal of ground has been covered in the paper under discussion.

M. GAMAL MOSTAFA,[1] A.M. ASCE.—The paper introduces a method of computation for the required live storage in a long-term storage reservoir which will guarantee a certain annual draft. In this field Hursts'[2,3] pioneer work has been widely acknowledge. The authors' line of approach yields, in general, smaller values for the required live storage.

For illustration of their method, the authors chose the case of Egypt's Sadd-el-Aali (Aswan High-Dam). They gave Fig. 13 for the relation between the necessary storage and the required draft for its reservoir.

The Sadd-el-Aali is a multiple use dam which will serve three major purposes; irrigation, flood control and production of power. Its final design is a rock-fill with an impervious core, horizontal blanket, vibrated dune sand and filter, grout curtain cut-off descending to bed rock, and a heavily reinforced concrete crest protector. The fèasibility of this design has long been established and approved by an International Board of Consultants in Nov. 1954, and by the Experts of The International Bank in 1955.

The major items in construction costs of the Sadd-el-Aali Project are the upstream cofferdam, the foundation, the diversion tunnels, and the hydroelectric power plants, all of which will not be affected by a small decrease in dam height. The authors suggest that a 45 milliards of live storage instead of the design 70 milliards would be enough for the Sadd-el-Aali reservoir. This would reduce the total capacity by 25 milliards, while saving only about 1% of the construction cost of the dam. However, a 45 milliards live storage may cause a great loss in agriculture if a series of low years such as those which actually happened in 1912, 13, and 1914, were to be repeated in the future—a matter which is quite probable. The minimum live storage which would guarantee a 70 milliards annual draft if no worse case of low years is provided for, should be equal to the total deficit in the supply during these years, which can be estimated as follows:

1. Chf. Engr., Sadd-el-Aali Dept., Cairo, Egypt.
2. Hurst, H. E. "Long-Term Storage of Reservoirs" Transactions ASCE, 1951.
3. Hurst, H. E. "Methods of Using Long-Term Storage in Reservoirs" The Institution of Civil Engineers, London, 1956.

		Natural River	Irrigation Demands	Reservoir Losses	Net
1912	Timely season	11	26	5	+20
	Untimely season	61	44	4	+13
1913	Timely season	9	26	4.5	-21.5
	Untimely season	35	44	3.5	-12.5
1914	Timely season	7	26	3	-22
				Total	-63 Mlrds.

The 63 milliards is therefore the minimum live storage that would meet a series of low years as 1912, 13 and 1914.

A trial regulation over a series of years similar to the past 54 years (1899-1953) taken in the order in which they had occurred revealed that a reservoir capacity of 70 milliards would have been needed to guarantee a steady annual flow equal to the mean. If, however, the sequence of occurrence in these years were changed, a matter which could not be ruled out as impossible, a regulation over the sequence 1899-1915, 1939-1953, 1916-1936, showed that for a steady annual flow equal to the mean a reservoir live capacity of 125 milliards would have been needed. For the series 1916-38, 1899-1915, 1939-1953, this figure was found to be 126 mlrds.

The seventy milliards live storage therefore contains no exaggeration but is a reasonable estimation of the necessary live storage for the Sadd-el-Aali reservoir.

Y. M. SIMAIKA,[1] and N. BOULOS[2].—The writers have read with particular interest the paper by Messrs. A. Fathy and Aly S. Shukry as they have both had a share in the investigations connected with the problem of reservoir capacity for long-term storage.

The authors took it as an assumption made by Hurst that when N = 2, $\frac{R}{\sigma} = 1$. Actually this is not an assumption, since for any two observations of a variable, it can be shown mathematically that $R = \sigma$.

Furthermore, on the remark made by the authors that formula (5) was not justifiable because $\frac{R}{\sigma} = 1$ when N = 2 was far from being satisfied in relation (2) of the random events "for which there was a theoretical foundation," the writers should comment here that relation (2) is not a purely mathematical one, but it is an approximation deduced by Hurst, Black and Simaika(7) based on some probability considerations and it is only near to the truth when N is a fairly big number. But when N is as small as 2, relation (2) should break down.

In the Nile Basin Vol. VII.,(7) sixty separate phenomena in about 90 series including river discharges, rainfall, temperature and pressure from stations scattered all over the world were dealt with indiscriminately. The

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observations were fairly represented by the equation:

$$\frac{R}{\sigma} = 1.65 \sqrt{N} \tag{27}$$

which is practically the same as equation (15) deduced by the authors.

In the attempt to settle the relation between R, σ and N more long term records of rainfall, temperature, atmospheric pressure, lake levels, annual growth of tree rings, varves, etc., were considered by Hurst who found that in all cases there was a linear relation between log $\frac{R}{\sigma}$ and log N. About 700 separate values of k were computed and it was found that individual values have a distribution which is approximately normal. The extreme values of k were 0.46, and 0.96 and the average 0.73 $\pm$ 0.006.

The procedure adopted by the authors in order to arrive at equation (15) is, in the opinion of the writers, another proof of the uniformity of character of physical phenomena.

In page 7 the authors thought it "inadvisable to rely on formulae derived through taking averages for a large number of phenomena." In this respect the writers may refer to the test mentioned by one of them in the contribution to the discussion of Hurst's "Methods of Using Long-Term Storage in Reservoirs"(8). p. 557. The test consisted in dividing long series of physical phenomena into periods of equal duration. A prediction for k was made from the former subperiod of the phenomenon and another prediction made from the average for all physical data (k = 0.73). In 16 out of 19 individual physical phenomena, the prediction made from the average proved nearer to the actual than the prediction made from the former subperiod of the same phenomenon. When all the 184 subdivisions were dealt with, the probable error of a single observation of k from the average was $\pm$ 0.060 whilst the probable error of those derived from the preceding periods was $\pm$ 0.075.

On the question of the capacity S, the principal suggestion made by the authors to deal with each phenomenon independently from all other phenomena is misleading since the value of S will depend to a big extent on the order of the record of observations of the phenomenon. It is clear that the order will not be repeated. A test for this treatment would be to divide a long-term record to subperiods and compare S predicted by such method from former subperiods with actual value of S. The test is done on the Aswan Discharges 1870 to 1956, 87 years of record. Those are divided into three equal subperiods. The maximum deficit S is calculated for various values of the draft B which is taken to include losses. Predictions of S called S_2 corresponding to various values of the draft are deduced from former subperiods and the percentage deviation from the actual is calculated.

Another prediction is made according to Hurst's formulae

$$\frac{R}{\sigma} = \left(\frac{N}{2}\right)^{0.73} \tag{28}$$

and

$$\frac{S}{R} = 0.94 - 0.96 \sqrt{(M - B)/\sigma} \tag{29}$$

which are slightly different from (6) and (8) quoted by the authors as the former represent the means for all natural events while the latter refer to

special groups of natural events. The difference is practically insignificant in the comparison. All the information for any prediction is taken from the immediately preceding subperiod.

Predicted values of storage S (called S_1) and their percentage deviations from the actual are calculated in Table IV.

Such tests give an idea of the probable deviation of forecasts from what will occur and at the same time make a fair comparison between the two methods of the prediction. The results of the test are:

1) For the second subperiod 1899 to 1927.
   Both predictions, S_1 and S_2 are zero for any draft below the mean of the period; consequently our method fails to compare the two predictions.
2) For the third subperiod 1928 to 1956.
   The predictions S_1 are nearer than S_2 to the actual in four out of the six calculated stages. In two stages S_2 is nearer than S_1;
   The average deviation of S_1 from $S = +3\%$ and the average deviation of $S_2 = +38\%$.

Another test of the same nature is made with the observations of New York rainfall 1826 to 1945, a record of 120 years. The record is divided into two periods of 60 years each. The computation for the test is shown in (Table V).

The results of the New York rainfall test are:

For the second subperiod 1885 to 1945.
The predictions S_1 are nearer than S_2 to the actual in all the 9 cases considered.
The average deviation of S_1 from the actual $= +102\%$, while the average deviation of S_2 from the actual $= +256\%$.

In the case of the proposed High Dam at Aswan it would therefore be unwise to ignore Hurst's equations which, up to the present time are the best means at our disposal for long term predictions of R and S.

Capacity of the High Dam at Aswan

The authors state that the 70 milliards assigned by the High Dam authority to the live storage is too large and claim (p. 27) that the most economical capacity is 45 milliards. We do not agree with the authors in their conclusion as it is based on one single phenomenon and not on the average of a large number of phenomena. The function of the High Dam will be three fold:

a) To guarantee the biggest annual quota to meet the irrigation requirements in the future.
b) To produce the biggest possible hydro-electric power.
c) To protect Egypt against the danger of the highest floods thus reducing the spilling of clear water and the possible degradation downstream.

All of these demand that the reservoir should be as big as possible within the limit when the increase of the extra losses will be more than balanced by the increase of the quota. Hurst and Black found that the optimum of the reservoir would be a live capacity of 70 milliards which would guarantee an annual quota of 82 milliards including Sudan requirements and unavoidable losses, and that an increase of 1 milliard in the quota could be produced by

TABLE IV.

PREDICTIONS OF S.

ASWAN DISCHARGES IN MILLIARDS OF CUBIC METRES.

Draft D	Maximum Deficit (actual) S	Prediction according to Hurst's formulae S_1	$\frac{S_1 - S}{S}$ %	Prediction without formulae S_2	$\frac{S_2 - S}{S}$ %
1. 1870 to 1898; N = 29; M = 110; σ = 13.8; R = 97					
108	78	No former information			
104	42				
100	24				
96	16				
82	0				
2. 1899 to 1927; N = 29; M = 83; σ = 14.1; R – 62					
82	57	0	– 100	0	– 100
80	49	0	– 100	0	– 100
78	42	0	– 100	0	– 100
76	36	0	– 100	0	– 100
74	31	0	– 100	0	– 100
72	28	0	– 100	0	– 100
3. 1928 to 1956; N = 29; M = 84; σ = 10.7; R = 72					
82	56	69	+ 23	57	+ 2
80	43	50	+ 16	49	+ 14
78	30	37	+ 23	42	+ 40
76	24	26	+ 8	36	+ 50
74	20	18	– 10	31	+ 55
72	17	10	– 41	28	+ 65
		Average	+ 3%	Average	+ 38%

TABLE V.

PREDICTION OF S.

NEW YORK RAINFALL IN INCHES.

Draft B	Maximum Deficit (Actual) S	Prediction according to Hurst's formulae		Prediction without formulae	
		S_1	$\frac{S_1 - S}{S}$ %	S_2	$\frac{S_2 - S}{S}$ %
1. 1826 to 1885 N = 60; M = 41.5; σ = 6.8; R = 108					
41	95				
40.5	82				
40	70				
39.5	60				
39	50	No former information			
38.5	41				
38	34				
37.5	28				
37	23				
2. 1886 to 1945 N = 60; M = 42.5; σ = 5.7; R = 84					
41	48	57	+ 19	95	+ 98
40.5	36	46	+ 28	82	+ 128
40	25	39	+ 56	70	+ 180
39.5	17	33	+ 94	60	+ 253
39	14	28	+ 100	50	+ 257
38.5	11	23	+ 109	41	+ 272
38	9	20	+ 122	34	+ 278
37.5	6	16	+ 166	28	+ 366
37	4	13	+ 225	23	+ 475
		Average	+ 102%	Average	+ 256%

an additional 40 milliards in the capacity but any further increase would actually reduce the draft.(9)

It should be here stated that the reduction suggested by the authors in the live capacity of the reservoir to 45 milliards will reduce the cost by only a small amount since the main cost of the High Dam is in the construction of the cofferdam, tunnels and the foundation.

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7. The Nile Basin Vol. VII. The Future Conservation of the Nile, by Hurst, Black and Simaika. Physical Department Paper No. 51. S.O.P. Press, Cairo, 1946.

8. Methods of Using Long-Term Storage in Reservoirs, by H. E. Hurst. Excerpt Proc. Inst. Civ. Engineers London. Part I, Vol. V, Sept. 1956.

9. Report on Hydrological Investigations on How the Maximum Value of the Nile Water May Be Made Available for Development in Egypt and the Sudan, by H. E. Hurst and R. P. Black Sudd el Aali Authority. Misr Press Cairo, 1955.

Discussion of
"GRAPHICAL DETERMINATION OF WATER-SURFACE PROFILES"

by Francis F. Escoffier
(Proc. Paper 1114)

VEN TE CHOW,[1] A.M. ASCE.—In the method described by the author, a curve of the varied flow function is plotted for a given hydraulic exponent n and the Bakhmeteff equation is solved graphically. In this procedure, a graphical interpolation is employed, and thus a direct solution of the depth of water at the end of a reach of prescribed length is made possible. This method replaces a trial and error procedure which is required in an algebraic solution of the Bakhmeteff equation.

In the Bakhmeteff method,(1) the change of the critical slope within a small range of the varying depth in each reach is assumed constant. If the critical slope changes from one range to the other in a given reach in an appreciable amount, values of σ, β, and w should no longer be assumed constant except for a very approximate solution. Consequently, the graphical procedure which requires the construction of a straight line having a slope equal to w becomes inapplicable. In such a case, it is suggested that η be plotted against wB in rectangular coordinates as shown in Fig. D-1. Thus, a given value of η - wB will appear in the resulting diagram as a straight line having a slope angle equal to 45^o. As shown in Fig. D-1, the value of I_{12} is represented by the vertical displacement between the two lines representing cross sections 1 and 2.

In a method described by the writer,(2) the variation in values of σ, β, and w is automatically implied in the surface-profile equation. This equation may be expressed as follows:

$$I_{12} = [\eta_2 - F(\eta_2)] - [\eta_1 - F(\eta_1)]$$

in which

$$F(\eta) = B + (y_c/y_o)^m (j/n)B_o$$

in which m is a hydraulic exponent such that the critical discharge varies with $y^{m/2}$, $j = n/(n - m + 1)$, and B_o is the varied flow function in which η is replaced by $\eta^{n/j}$ and n by j. The graphical procedure suggested by the author is equally applicable to the writer's method by plotting η against $F(\eta)$ as shown in Fig. D-1.

In applying the graphical procedure to channels of adverse slope, the author has recommended the use of a table published by Matzke.(2) It is to be noted that Matzke's table covers a range of n from 3.0 to 4.0. For broadening the scope of application, the writer has prepared a table(D-1) which covers a wider range of n from 2.0 to 5.5.

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It may be also noted that a chart similar to Fig. 8 in the paper has been published by the writer(7) with the depth-base ratio and depth-diameter ratio plotted in a logarithmic scale.

The author is to be complimented for introducing a time-saving procedure for the determination of water-surface profiles.

REFERENCE

D-1. Ven Te Chow, Discussion of Integrating the Equation of Gradually Varied Flow, Proc. A.S.C.E., Vol. 81, Paper No. 838, 32 pp, Nov. 1955.

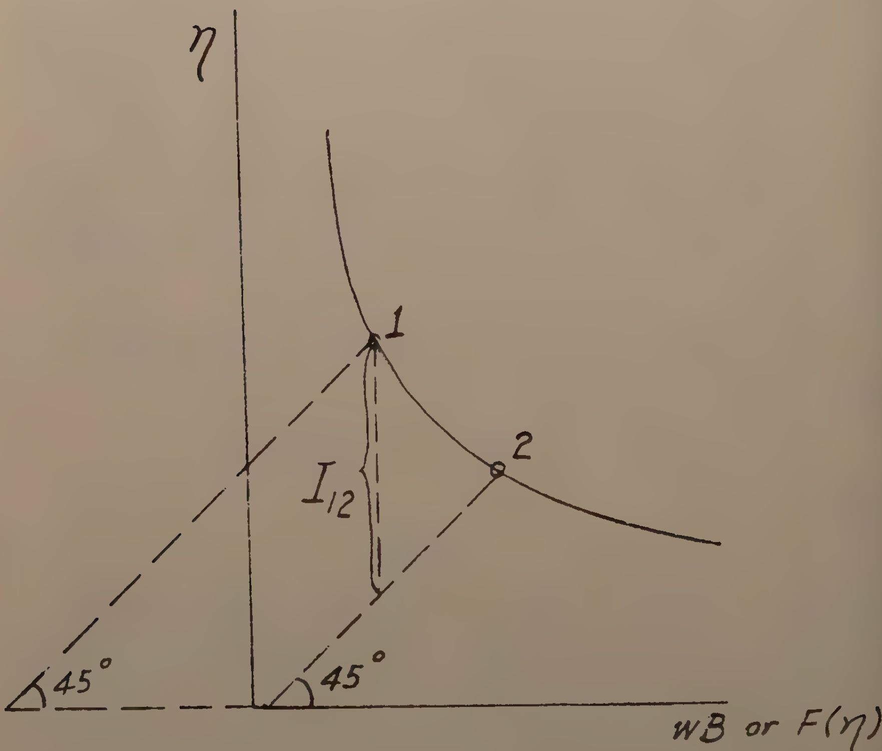

Fig.D-1 A Modified Graphical Procedure

Discussion of
"BUTTERFLY VALVE FLOW CHARACTERISTICS"

by M. B. McPherson, H. S. Strausser, and J. C. Williams
(Proc. Paper 1167)

CORRECTIONS.—On page 1167-14, the caption for Fig. 7 should read "Continuous Pipe (Plot of Data from Table IV)." On page 1167-20, line 5, the sentence "With this assumption, it would . . . of separation" should be deleted. On page 1167-21, in the line above Eq. (7-c), the quantity in parentheses should be $(V_2 - V_3)$ rather than $(V_2 = V_3)$.

PROCEEDINGS PAPERS

e technical papers published in the past year are identified by number below. Technical-on sponsorship is indicated by an abbreviation at the end of each Paper Number, the ls referring to: Air Transport (AT), City Planning (CP), Construction (CO), Engineering nics (EM), Highway (HW), Hydraulics (HY), Irrigation and Drainage (IR), Power (PO), ry Engineering (SA), Soil Mechanics and Foundations (SM), Structural (ST), Surveying and ng (SU), and Waterways and Harbors (WW) divisions. Papers sponsored by the Board of ion are identified by the symbols (BD). For titles and order coupons, refer to the appro- issue of "Civil Engineering." Beginning with Volume 82 (January 1956) papers were ned in Journals of the various Technical Divisions. To locate papers in the Journals, the ls after the paper numbers are followed by a numeral designating the issue of a particular l in which the paper appeared. For example, Paper 1113 is identified as 1113 (HY6) indicates that the paper is contained in the sixth issue of the Journal of the Hydraulics on during 1956.

VOLUME 82 (1956)

L: 925(WW2), 926(WW2), 927(WW2), 928(SA2), 929(SA2), 930(SA2), 931(SA2), 932(SA2)[c], (SM2), 934(SM2), 935(WW2), 936(WW2), 937(WW2), 938(WW2), 939(WW2), 940(SM2), 941 12), 942(SM2)[c], 943(EM2), 944(EM2), 945(EM2), 946(EM2)[c], 947(PO2), 948(PO2), 949(PO2), (PO2), 951(PO2), 952(PO2)[c], 953(HY2), 954(HY2), 955(HY2)[c], 956(HY2), 957(HY2), 958 2), 959(PO2), 960(PO2).

961(IR2), 962(IR2), 963(CP2), 964(CP2), 965(WW3), 966(WW3), 967(WW3), 968(WW3), 969 V3), 970(ST3), 971(ST3), 972(ST3)[c], 973(ST3), 974(ST3), 975(WW3), 976(WW3), 977(IR2), (AT2), 979(AT2), 980(AT2), 981(IR2), 982(IR2)[c], 983(HW2), 984(HW2), 985(HW2)[c], 986(ST3), (AT2), 988(CP2), 989(AT2).

990(PO3), 991(PO3), 992(PO3), 993(PO3), 994(PO3), 995(PO3), 996(PO3), 997(PO3), 998 3), 999(SA3), 1000(SA3), 1001(SA3), 1002(SA3), 1003(SA3)[c], 1004(HY3), 1005(HY3), 1006 3), 1007(HY3), 1008 (HY3), 1009 (HY3), 1010 (HY3)[c], 1011 (PO3)[c], 1012 (SA3), 1013 (SA3), 4(SA3), 1015(HY3), 1016(SA3), 1017(PO3), 1018(PO3).

1019(ST4), 1020(ST4), 1021(ST4), 1022(ST4), 1023 (ST4), 1024(ST4)[c], 1025(SM3), 1026 3), 1027(SM3), 1028(SM3)[c], 1029(EM3), 1030(EM3), 1031(EM3), 1032(EM3), 1033(EM3)[c].

ST: 1034(HY4), 1035(HY4), 1036(HY4), 1037(HY4), 1038(HY4), 1039(HY4), 1040(HY4), 1(HY4)[c], 1042(PO4), 1043(PO4), 1044(PO4), 1045(PO4), 1046(PO4)[c], 1047(SA4), 1048 4)[c], 1049(SA4), 1050(SA4), 1051(SA4), 1052(HY4), 1053(SA4).

MBER: 1054(ST5), 1055(ST5), 1056(ST5), 1057(ST5), 1058(ST5), 1059(WW4), 1060(WW4), 1(WW4), 1062(WW4), 1063(WW4), 1064(SU2), 1065(SU2), 1066(SU2)[c], 1067(ST5)[c], 1068 V4)[c], 1069(WW4).

BER: 1070(EM4), 1071 (EM4), 1072(EM4), 1073(EM4), 1074(HW3), 1075 (HW3), 1076(HW3), 7(HY5), 1078(SA5), 1079(SM4), 1080(SM4), 1081(SM4), 1082(HY5), 1083(SA5), 1084(SA5), 5(SA5), 1086(PO5), 1087(SA5), 1088(SA5), 1089(SA5), 1090(HW3), 1091(EM4)[c], 1092 5)[c], 1093(HW3)[c], 1094(PO5)[c], 1095(SM4)[c].

MBER: 1096(ST6), 1097(ST6), 1098(ST6), 1099(ST6), 1100(ST6), 1101(ST6), 1102(IR3), 1103), 1104(IR3), 1105(IR3), 1106(ST6), 1107(ST6), 1108(ST6), 1109(AT3), 1110(AT3)[c], 1111(IR3)[c], ST6)[c].

MBER: 1113(HY6), 1114(HY6), 1115(SA6), 1116(SA6), 1117(SU3), 1118(SU3), 1119(WW5), (WW5), 1121(WW5), 1122(WW5), 1123(WW5), 1124(WW5)[c], 1125(BD1)[c], 1126(SA6), 1127), 1128(WW5), 1129(SA6)[c], 1130(PO6)[c], 1131(HY6)[c], 1132(PO6), 1133(PO6), 1134(PO6), (BD1).

VOLUME 83 (1957)

RY: 1136(CP1), 1137(CP1), 1138(EM1), 1139(EM1), 1140(EM1), 1141(EM1), 1142(SM1), (SM1), 1144(SM1), 1145(SM1), 1146(ST1), 1147(ST1), 1148(ST1), 1149(ST1), 1150(ST1), (ST1), 1152(CP1)[c], 1153(HW1), 1154(EM1)[c], 1155(SM1)[c], 1156(ST1)[c], 1157(EM1), 1158 1), 1159(SM1), 1160(SM1), 1161(SM1).

ARY: 1162(HY1), 1163(HY1), 1164(HY1), 1165(HY1), 1166(HY1), 1167(HY1), 1168(SA1), (SA1), 1170(SA1), 1171(SA1), 1172(SA1), 1173(SA1), 1174(SA1), 1175(SA1), 1176(SA1), (HY1)[c], 1178(SA1), 1179(SA1), 1180(SA1), 1181(SA1), 1182(PO1), 1183(PO1), 1184(PO1), (PO1)[c].

1186(ST2), 1187(ST2), 1188(ST2), 1189(ST2), 1190(ST2), 1191(ST2), 1192(ST2)[c], 1193), 1194(PL1), 1195(PL1).

1196(EM2), 1197(HY2), 1198(HY2), 1199(HY2), 1200(HY2), 1201(HY2), 1202(HY2), 1203), 1204(SM2), 1205(SM2), 1206(SM2), 1207(SM2), 1208(WW1), 1209(WW1), 1210(WW1), (WW1), 1212(EM2), 1213(EM2), 1214(EM2), 1215(PO2), 1216(PO2), 1217(PO2), 1218), 1219(SA2), 1220(SA2), 1221(SA2), 1222(SA2), 1223(SA2), 1224(SA2), 1225(PO)[c], 1226 1)[c], 1227(SA2)[c], 1228(SM2)[c], 1229(EM2)[c], 1230(HY2)[c].

ussion of several papers, grouped by Divisions.